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Application of strength criteria in describing modified wood

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Abstract:
The study was carried out on a series of 360 samples of Scots pine wood (Pinus sylvestris). A part of them were in their natural state, but the majority of them were superficially polymerized with poly(methyl acrylate). The test specimen comprised natural and modified wood with varying poly(methyl acrylate) (PMM) contents. The samples were tested for tensile strength on a universal testing machine. The purpose of the experiment was to examine the ways, in which polymerization improves on the strength properties and how the properties of materials change if the angle α between the load direction and the fiber orientation changes. An additional purpose of the study was to select an adequate strength criterion to describe the tested composite materials. Samples made of modified wood were uniaxially stretched at an angle of $\alpha = 0, 15, 30, 45, 60$ and 90^{0}, measured between the direction of the load and the longitudinal direction of the fibers.

It was claimed that the higher the polymer contents, the better the strength properties of modified wood.

The study proposed its own model, which describes the bending stiffness of modified wood.

***Key Word**: natural wood, modified wood, static tensile test, strength criteria.*

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I. Introduction

A number of methods has been developed with the purpose of increasing durability of wood as structural material. One of them is impregnation with a monomer and subsequent polymerization in situ. The resulting material is termed wood polymer composite (WPC). There are two types of composites obtained in this way: cell lumen type and cell wall type [1]. Cell lumen impregnation of wood creates a material in which the polymer fills the wood cell cavities, which increases the stability of the internal structure of the wood.

Such modification results in a material with higher resistance to crushing and higher overall stiffness and hardness [1–5]. In the case of extension the polymer filling the cell cavity decreases freedom of deformation of the entire cell, whereas in the case of compression it retards buckling of the cell walls oriented parallel to the external compressing forces. The fuller the polymer fills the cavities, the more pronounced this reinforcing effect is. Tensile strength, although it provides merely rough characteristics of a given material's mechanical properties, is commonly used in industry due to the ease of conducting and relatively low cost of the tensile test comparing to other, more precise, tests. Out of voluminous literature on mechanical properties of wood and WPC only a few articles treat tensile strength, and among those that do most cover tensile of natural, untreated wood. Research on this property of wood can be traced back to the prewar times [6] for tensile strength of birch and for earlier references and already then effort was directed towards establishing relation between tensile and density. In recent times some revival of interest in this topic is noticeable.

The subject-matter of this work is strength of sapwood obtained from Scots pine (Pinus sylvestris) and the WPCs manufactured by its polymerization with the use of methyl methacrylate (MMA). Scots pine is ubiquitous throughout Northern Europe and is commonly used as a relatively cheap and easily available construction material. Its application ranges from civil engineering through port and harbor engineering to broadly understood shipping.

It is commonly used as a material for dock fenders and for the upper layer of the keelblocks where ship's bottom comes into contact with the support. Combined action of seasonal, cyclic variation of temperature and moisture, especially when these members work within the water-air interface, leads to their accelerated deterioration and eventually damage. The wintertime is particularly critical in this respect, since below-zero temperatures can cause bursting of internal wood structure. Besides, fenders fastened to the wharfs are subject to impact loads while mooring and cargo handling, which often results in their mechanical damage. This work was motivated by possible application of WPC in a broadly understood marine industry, where materials are exposed to the action of unfavorable environmental factors. Assuming that strength provides valuable macroscopic index of usefulness of the material, an experiment was designed to assess the necessary level of impregnation with polymer form the point of view of specific needs and to investigate how content the polymer and load direction influenced on the strength. The goal was to obtain quantitative material suitable for further analysis and planning development of research.

In the majority of studies, authors focus on wood strength depending on the angle α between the direction of the load and the direction of wood fibers , or on polymer content only. Studies ref. [1, 4, 5, 7 - 12] present the interrelation between the properties of modified wood, such as tensile strength/compressive strength along and across the fibers, Young's modulus, and the quantity of modifying substance, with which the wood was impregnated. All changes of mechanical properties of wood, induced by modifications, cannot be predicted without performing specific tests.

The strength of a structure exposed to external loads is analyzed on the basis of the status of the materials used to construct it. The basic strength criteria applicable to isotropic materials include, among others, the criteria adopted by Galileo, Lamie, Clebsch, Mariotte, Navier, Coulomb, Maxwell, Gienk. In turn, strength criteria applicable to anisotropic materials include those adopted by Goldenblat - Kopnov, Kowalczuk, Lebiedev, Pisarenko, Fiszer, Hill, Tsai - Wu and others [13 - 17]. An analysis of strength criteria used to describe orthotropic materials [13 - 43] has indicated that there is no universal strength criterion used to describe anisotropic materials, which is why selected criteria are applied to specific materials, analyzing the results obtained. An analysis of test results obtained from the model tests was presented, among others, in studies ref. [31, 32, 43].

Studies of the strength of materials and structures generally apply the Goldenblat - Kopnov yield criterion, the Tsai-Wu criterion and the Hoffman criterion [20, 28, 29, 30, 32, 33]. Among recent developments, the most notable criteria are presented in studies ref. [35 - 37]. Wood is cellular and is therefore classified as an anisotropic material. In result, the actual orientation of the "fibers" (usually referred to as the orientation of all axial cellular elements) inside a piece of wood will have a strong impact on its mechanical properties [44]. Structural wood is usually longitudinally used, and the majority of wood pieces are characterized by angular fiber positioning in the figure. Even in the case of trees with relatively straight fibers, structural wood often has trunk faults, whereby they are not actually cylindrical. What is more, fibers are rarely ideally straight in trees and deviations are frequent, particularly in coniferous trees and in some hard wood [45].

Wood is characterized by anisotropic properties with strength asymmetry. It is therefore important to project the loads relative to non-parallel fiber orientations. In the study, the strength properties of wood were described according to known strength criteria applicable to anisotropic materials, i.e. the von Mises yield criterion and the Aśkenazi criterion. Furthermore, two additional strength criteria were proposed by the author to describe superficially modified wood.

Wood is classified as anisotropic material, which makes its description difficult. There are three principal directions due to existence of three planes of symmetry: the radial – R (x_1), the tangential – T (x_2) and the longitudinal – L (x_3) – Fig. 1. If a sample is cut out sufficiently far away from the center, wood can be treated as an orthotropic material [4, 11].

II. Material and test procedure

An important issue is to determine the impact of poly(methyl acrylate) (PMM) contents in the composite on the tensile strength properties, and to determine how a change of the angle α between the direction of the load and the direction of the fibers affects its strength.

For this purpose, samples were collected from an angular boulder without any defects in the form of knags, rot, etc. Wood boulders were seasoned and naturally dried at (22 ± 2^0 C) and at relative humidity of 75% in laboratory conditions. Samples for testing were taken from an angular boulder of (240x120x10) (LxTxR) - anatomical wood dimensions (x_1, x_2,x_3)- coordinates used in the elasticity theory.

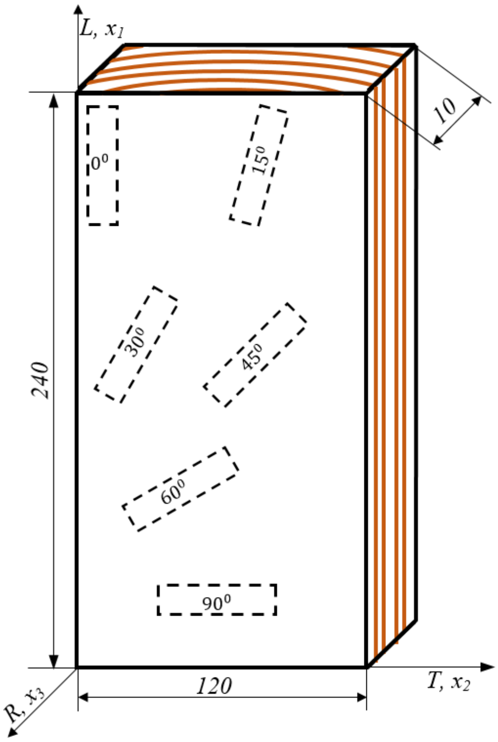

Fig. 1: Method of sampling the angular boulder tensile for strength testing depending on the angle α of fibers in reference to the longitudinal

The samples created retained the orientation of their length at the angle of α=0,15,30,45,60,90^0 in reference to the fiber longitudinal (Fig. 1). The tests conducted aimed to determine the tensile strength of modified wood, depending on the polymer content in the composite and the angle (α=0,15,30,45,60,90^0) between the direction of the load applied and the fiber longitudinal.

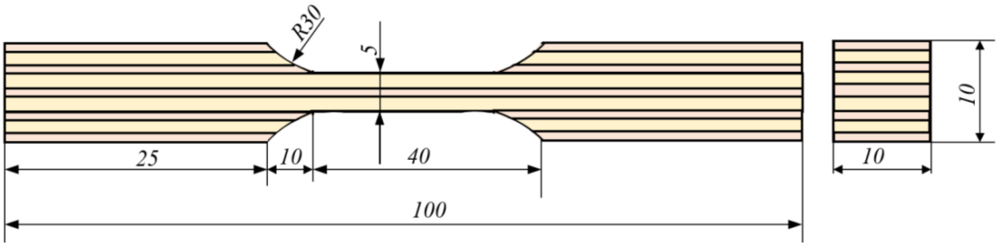

Fig. 2: Shape and dimensions of samples for static tensile testing

The shape and dimensions of the samples are presented in Fig. 2. The average humidity of natural wood samples was 12 - 15%. The samples were divided into two groups. The first group was made of natural wood, whereas the second group was made of modified wood. Superficial wood modification is a process which consists in saturating natural wood with methyl methacrylate (MM) stabilized with hydroquinone methylether, and then performing thermal polymerization. The course of the superficial wood modification process is presented in studies ref. [2, 4, 10 - 12]. Correctly prepared samples were stretched in a universal testing machine.

The results pertaining to the strength of natural wood K0.0 and modified wood K0.35-K0.56 were determined for each angle α in 10 repetitions.

Table no 1: Strength properties of composite, determined on samples subject to tensile strength testing.

Material type	Polymer content, %	Angle between the direction of the force applied and the fibers, ^{0}					
K0.0	-	0	15	30	45	60	90
		R_m, MPa					
		92	62	42	25	16	5
		97	54	41	25	17	4
		95	60	37	23	16	4

		98	53	40	22	16	5
		94	58	38	25	18	4
		98	54	36	25	18	4
		93	62	35	26	15	5
		95	56	39	26	16	5
		92	63	37	24	16	4
		96	58	38	26	16	5
K0.35	35	103	76	58	40	29	5
		101	76	54	39	25	5
		100	75	56	40	24	7
		105	77	61	40	25	5
		102	74	55	38	25	6
		99	72	55	36	25	9
		103	73	48	36	29	5
		106	71	56	37	31	7
		100	72	52	36	27	8
		102	74	50	38	25	6
K0.43	43	110	78	62	39	30	7
		107	79	60	42	31	8
		106	84	59	43	30	8
		108	79	60	40	28	8
		110	79	62	43	28	8
		112	76	64	46	26	6
		114	74	65	43	28	8
		113	77	60	44	25	8
		111	76	66	47	26	8
		110	78	62	43	28	7
K0.48	48	115	82	65	47	33	8
		110	84	60	52	30	8
		108	80	64	45	32	8
		112	78	65	49	25	8
		109	82	66	48	32	7
		115	83	67	47	35	8
		112	86	68	51	36	8
		114	84	72	44	34	9
		113	79	67	49	32	8
		112	82	66	48	31	8
K0.56	56	115	89	70	57	40	8
		120	89	75	58	35	9
		116	93	70	59	42	10
		118	84	72	54	32	8
		112	88	71	55	38	9
		123	92	68	56	46	9
		121	81	72	51	34	9
		116	87	69	52	41	10
		121	87	72	53	36	10
		118	90	71	55	38	8

In K0.0 - K0.56 the numbers indicate the amount of PMMA in kilograms per 1kg of wood with a moisture level of 8% [2].

Table no 2: Presents statistical parameters of static tensile testing of natural and modified wood.

Material type	Parameter statistics	α, ^{0}					
		0	15	30	45	60	90
Natural wood K0.0	$\overline{R}_m$,MPa S_n,MPa V,% K_1	95.0 2.14 2.2 1.40	58.0 3.49 6.0 1.43	38.0 3.87 10.1 1.03	25.0 4.77 19.1 0.62	16.4 1.91 11.6 0.83	4.5 0.5 11.1 1.00
Modified wood K0.35	$\overline{R}_m$,MPa S_n,MPa V,% K_1	102.0 2.58 2.7 1.51	74.0 3.63 4.9 0.82	54.5 4.94 9.0 1.31	38.0 3.40 8.9 0.59	26.5 3.42 12.9 1.31	6.3 1.49 23.6 1.81
Modified wood K0.43	$\overline{R}_m$,MPa S_n,MPa	110.0 5.88 5.3 0.68	78.0 3.37 4.3 1.78	62.0 5.88 9.5 0.68	43.0 3.87 9.0 1.03	28.0 2.56 9.1 1.17	7.6 1.31 17.2 1.22

	V,% K_1						
Modified wood K0.48	$\overline{R}_m$,MPa S_n,MPa V,% K_1	112.0 3.60 3.2 1.11	82.0 3.00 3.7 1.33	66.0 2.89 4.4 2.07	48.0 2.32 4.8 1.72	32.0 3.95 12.3 1.77	7.9 1.24 15.7 0.88
Modified wood K0.56	$\overline{R}_m$,MPa S_n,MPa V,% K_1	118.0 5.62 4.7 0.89	88.0 3.74 4.2 1.87	70.9 2.77 3.9 1.48	55.0 3.19 5.8 1.25	38.0 4.31 11.3 1.85	8.9 1.69 18.9 0.65

where:
Notation
α - angle between the direction of the load applied and the fiber longitudinal;
$\overline{R}_m$ - average tensile strength;
S_n - standard deviation;
V - variation coefficient;
K_1 - coefficient of elimination of gross errors;
P'_{ijkl} - coordinates of the strength surface tensor in the inverse system;
$\sigma'_{ij}, \sigma'_{kl}$ - coordinates of strength tensors in the inverse system;
α_{im} - cosine of the angle between the i-th axis in the new system and the m-th axis in the old coordinate system.

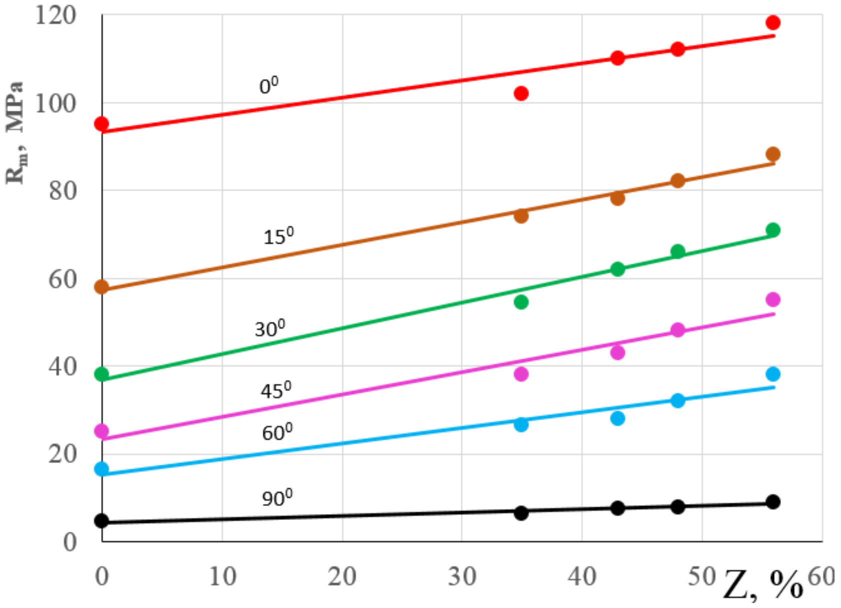

Fig. 3: Tensile strength $\overline{R}_m$ of modified wood from PMA z content for fixed values of angle α: points - experimental data, continuous lines - approximation of LSM experimental data

Fig. 3 presents the results of strength tests for wood composites which were superficially modified with methyl methacrylate. The points stand for averaged values taken from Table 2, whereas the straight lines were obtained using the Least Squares Method (LSM).

Description of tensile strength:

For isotropic materials, strength hypotheses have the following form [2, 43, 46, 47]:

$$F(\sigma_{ij})=C \qquad (1)$$

where:

the left-hand side is the stress function, whereas the right-hand side is the value obtained at uniaxial stress. If the left-hand side can be expressed in primary stress, it will be the function of three stresses.

For anisotropic materials, both the stress tensor components and certain tensor values characterizing the strength properties of the material should be taken into account. The general strength criterion for anisotropic materials is [2, 43, 46, 47]:

$$F(\pi_{ij}\sigma_{ij},\pi_{ijkl}\sigma_{ij}\sigma_{kl},.....)=0 \qquad (2)$$

where:

$\pi_{ij},\pi_{ijkl},...$ – tensors expressing anisotropic properties of the material;

$\sigma_{ij},\sigma_{ij}\sigma_{kl}$ – stress tensor values.

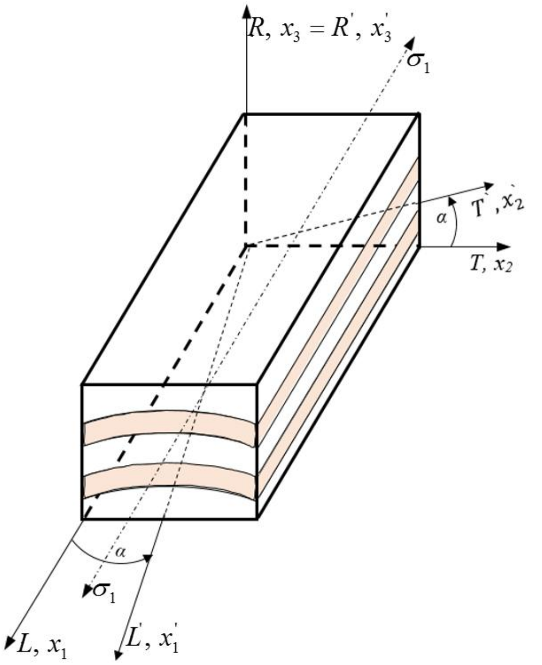

Fig. 4: Method of loading modified wood samples after rotating the coordinate system around axis x_3 by a preset angle of α

Fig. 4 presented the diagram of modified wood sample load after they were rotated by a preset angle of α in a coordinate system around axis x_3.

Among an abundance of known criteria used to describe the strength of anisotropic materials, the von Mises and Aśkenazi criteria were adopted in describing the results of experimental studies.

Von Mises yield criterion:

The von Mises yield criterion for orthotropic materials is expressed as [2, 43, 48]:

$$P'_{ijkl}\sigma'_{ij}\sigma'_{kl} = 1 \qquad (3)$$

where:

P'_{ijkl} - coordinates of the strength surface tensor in the inverse system;

$\sigma'_{ij},\sigma'_{kl}$ - coordinates of strength tensors in the inverse system.

Applying the fourth-order transformational formula for orthotropic material:

$$P'_{ijkl} = \alpha_{im}\alpha_{jn}\alpha_{ko}\alpha_{lp}P_{mnop},\ i,j,k,l=1,2,3 \qquad (4)$$

where:

α_{im} – cosine of the angle between the *i*-th axis in the new system and the *m*-th axis in the old system.

When rotated around axis $x_1 = x_1'$ (Fig. 4), the cosines take values according to table 3.

Table no 3: Cosines of angles between axes, when rotated around axis x_3

	x_1	x_2	x_3
x_1'	$\cos\alpha$	$\sin\alpha$	0

x'_2	$-\sin\alpha$	$\cos\alpha$	0
x'_3	0	0	1

Considering the values from table 4, the dependence (4) for $i = j = k = l = 1$ an ortotropic body has the form:

$$P'_{1111} = P_{1111}\cos^4\alpha + \left(2P_{1122} + 4P_{1212}\right)\cos^2\alpha\sin^2\alpha + P_{2222}\sin^4\alpha \tag{5}$$

The study assumes that $\sigma_{ij} = R_\alpha$.

Applying dependences (5) and (3), the following equations were obtained after substituting angles $\alpha = 0,15,90^o$:
(for α= 0,15,90^o , a negative value would be present under the root)

$$[P_{1111}\cos^4\alpha + \left(2P_{1122} + 4P_{1212}\right)\cos^2\alpha\sin^2\alpha + P_{2222}\sin^4\alpha] \cdot \sigma'^2_{11} = 1 \tag{6}$$

For $\alpha = 0^0$, based on dependence (6), considering that $\sigma'_{11} = R_0$:

$$P_{1111}R_0^2 = 1 \Rightarrow P_{1111} = \frac{1}{R_0^2} \tag{7}$$

for $\alpha = 90^0$, based on dependence (6), considering that $\sigma'_{11} = R_{90}$:

$$P_{2222}R_{90}^2 = 1 \Rightarrow P_{2222} = \frac{1}{R_{90}^2} \tag{8}$$

for $\alpha = 15^0$, based on dependence (6), considering that $\sigma'_{11} = R_{15}$:

$$2P_{1122} + 4P_{1212} = \left(\frac{1}{R_{15}^2} - P_{1111}\cos^4 15^0 - P_{2222}\sin^4 15^0\right)/(\cos^2 15^0 \sin^2 15^0) \tag{9}$$

Based on dependence (4), considering that $\sigma'_{11} = R_\alpha$:

$$R_\alpha = \frac{1}{\sqrt{P'_{1111}}}. \tag{10}$$

After substituting equations (7), (8), (9) in dependence (5), assuming that sin 15^0 =0.25, cos15^0 =0.966 and dependence (10), the following equation was obtained:

$$R_\alpha = \frac{R_0}{\sqrt{\cos^4\alpha + b_1\sin^2 2\alpha + c_1\sin^4\alpha}}. \tag{11}$$

For α =90^0 , the equation is:

$$c_1 = \left(\frac{R_0}{R_{90}}\right)^2,$$

whereas for angle α_1=15^0 , b_1 was determined as:

$$b_1 = \frac{1}{\sin^2 2\alpha_1}\left[\frac{R_o^2}{R_{\alpha_1}^2} - \cos^4\alpha_1 - \frac{R_o^2}{R_{90}^2}\sin^4\alpha_1\right]. \tag{12}$$

Table no 4: Contains the values of b_1 and c_1 coefficients determined on the basis of dependence (12).

Material type	Coefficients	
	b_1	c_1
K0.0	-0.752	445.7
K0.35	-0.588	262.1
K0.43	0.712	209.5
K0.48	0.374	201.0
K0.56	0.552	175.8

Table no 5: Selected average values of material strength, depending on the *Z* polymer content and angle *α*.

Polymer content, %	$R_{0,}$ *MPa*	$R_{15,}$ *MPa*	$R_{90,}$ *MPa*
0	95	58	4,5
35	102	74	6,3
43	110	78	7,6
48	112	82	7,9
56	118	88	8,9

Applying the LSM, the data from table 5 was used to determine the dependences which describe strength R_0,R_{15},R_{30},R_{45},R_{60} and R_{90} relative to PMM content z in the form of:

$$R_0 = 93.2 + 0.39z,\ \ r = 0.8901 \tag{13a}$$

$$R_{15} = 57.2 + 0.52z,\ \ r = 0.9848 \tag{13b}$$

$$R_{30} = 36.9 + 0.58z,\ \ r = 0.9817 \tag{13c}$$

$$R_{45} = 23.4 + 0.50z,\ \ r = 0.9459 \tag{13d}$$

$$R_{60} = 15.3 + 0.35z,\ \ r = 0.9314 \tag{13e}$$

$$R_{90} = 4.27 + 0.076z,\ \ r = 0.9484 \tag{13f}$$

where:
z – PMA content in %, r – correlation coefficient (critical value of this coefficient according to dependence (13) for significance level of 0.05 and at 3 degrees of freedom (0.878). In dependence (13), the values of R_0,R_{15},R_{30},R_{45},R_{60} are expressed in MPa.

Dependence (11) and the values of coefficients from table 4 were then used to calculate the strength of the composite, depending on polymer content z and angle α. The results of the calculations are presented in Table 6 and in Fig. 4.

Table no 6: The results of material strength calculations depending on PMA content z and angle α based on the von Mises yield criterion (11).

Composite	Calculated strength R_α, MPa for α, ^{0}					
	0	15	30	45	60	90
K0.0	95	58	18.0	9.0	6.0	4.5
K0.35	102	74	25.1	12.6	8.4	6.3
K0.43	110	78	29.2	15.1	10.1	7.6
K0.48	112	82	30.6	15.7	10.5	7.9
K0.56	118	88	34.1	17.6	11.8	8.9

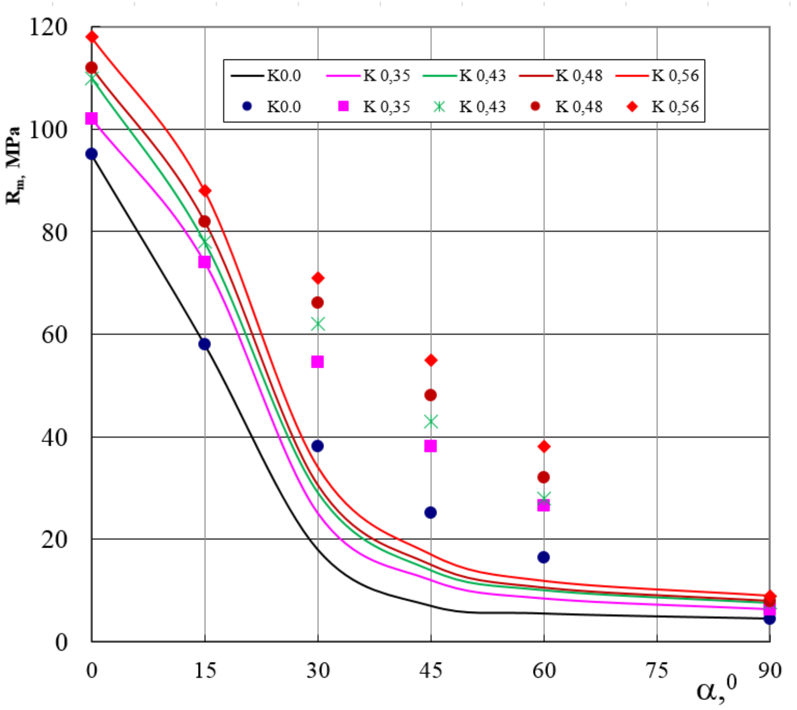

Fig. 5: The results of tensile tests of wood composite K.0÷K0.56: points - experimental data, lines - according to the von Mises criterion (11)

Fig. 5 presents the average results of experimental studies and theoretical curves obtained by applying the von Mises criterion (11). The von Mises criterion applied was found to insufficiently describe experimental data. Therefore, other criteria were adopted in describing the strength properties of tested composite materials.

Aśkenazi criterion:
Aśkenazi formulated a criterion for anisotropic materials in the form of [2, 49]:

$$a'_{ijkl}\sigma'_{ij}\sigma'_{kl} - \sqrt{\frac{\left(\sigma'_{ii}\right)^2 + \sigma'_{ik}\sigma'_{ik}}{2}} = 0 \qquad (14)$$

Applying similar reasoning to von Mises, the following equation was obtained:

$$R_\alpha = \frac{R_0}{\cos^4\alpha + b\sin^2 2\alpha + c\sin^4\alpha}\,, \qquad (15)$$

for any α_1:

$$\cos^4\alpha_1 + b\sin^2 2\alpha_1 + c\sin^4\alpha_1 = \frac{R_0}{R_{\alpha_1}}\,;$$

for $\alpha = 90^0$, $R_{90} = \frac{R_0}{c}$, $c = \frac{R_0}{R_{90}}$, after transformation

$$b\sin^2 2\alpha_1 = \frac{R_0}{R_{\alpha_1}} - \cos^4\alpha_1 - c\sin^4\alpha_1\,.$$

Hence:

$$b = \frac{1}{\sin^2 2\alpha_1}\left(\frac{R_0}{R_{\alpha_1}} - \cos^4\alpha_1 - c\sin^4\alpha_1\right)$$

where:

$$c = \frac{R_0}{R_{90}}\,,\ b = \frac{1}{\sin^2 30^0}\left[\frac{R_0}{R_{15}} - \cos^4 15 - c\sin^4 15\right]. \qquad (16)$$

In order to compare the Aśkenazi criterion with the von Mises criterion, the same angles of $\alpha=0^0$, 15^0 and 90^0 were adopted in calculations.

Table no 7: Calculated *b* and *c* coefficients determined on the basis of dependence (16)

Composite	Coefficients	
	b	*c*
K0.0	2.689	21.111
K0.35	1.738	16.19
K0.43	1.896	14.474
K0.48	1.726	14.177
K0.56	1.642	13.258

The values calculated on the basis of dependence (13) were substituted in dependence (16) and the values of *b* and *c* coefficients were calculated, as given in table 7. Dependence (15) and the values of coefficients from Table 7 were then used to calculate the strength of the composite, depending on polymer content *z* and angle *α*. The results of the calculations are presented in Table 8 and in Fig. 6.

Table no 8: The results of material strength calculations depending on PMA content *z* and angle *α*.

Composite	Calculated strength R_α, *MPa* for α, ^{o}					
	0	15	30	45	60	90
K0.0	95	58	24.4	11.6	6.8	4.5
K0.35	102	74	35.4	16.9	9.7	6.3
K0.43	110	78	38.1	19.1	11.4	7.6
K0.48	112	82	40.8	20.3	12.0	7.9
K0.56	118	88	45.0	22.7	13.5	8.9

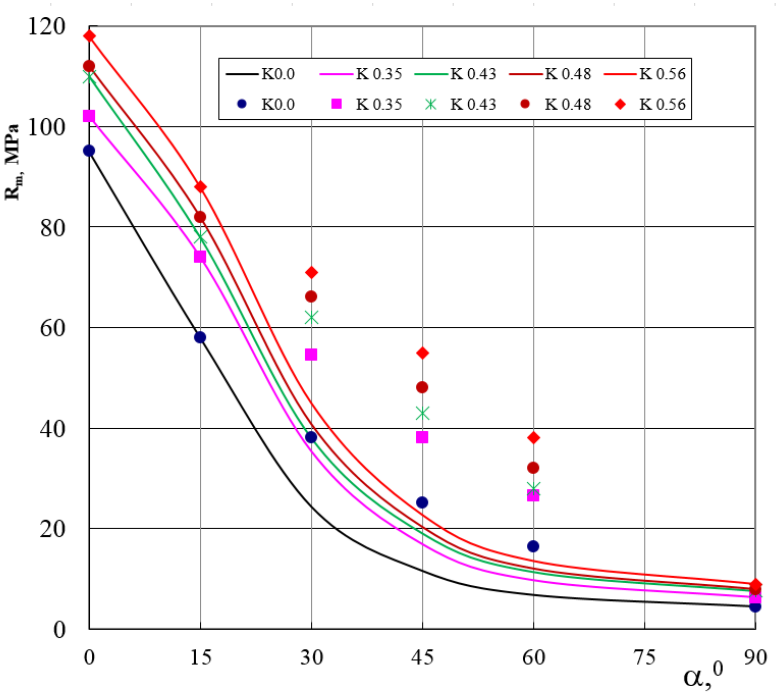

Fig. 6: The results of tensile tests of wood composite K.0÷K0.56 : points - experimental data, lines - according to the Aśkenazi criterion (15)

Fig. 6 presents the average results of experimental studies and theoretical curves obtained from dependence (15). The Aśkenazi was found to describe experimental data slightly better than the von Mises criterion. However, the results were still unsatisfactory. Therefore, an attempt was made to describe the strength parameters of modified wood by applying a linear and square function.

Strength description *R=R(α)* applying the linear function:

The strength characteristics presented in Figs. 5 and 6 based on von Mises and Aśkenazi criteria are far from satisfactory. This means that the adopted strength criteria insufficiently describe experimental data for composites, depending on polymer content z and angle α . Since the course of experimental data is near-linear, a linear function was proposed to describe them:

$$R_\alpha = b\alpha + a \,. \tag{17}$$

Due to small values of strength for angle α=90^0 , to make sure negative values are not obtained from dependence (17), a straight line crossing an empirical point for α=90^0 , was adopted. Then, the straight line formula has the following form:

$$R_\alpha = R_{90} + A(90 - \alpha), \tag{18}$$

The coefficient R_{90} expressing the strength of composites K.0÷K0.56 for α=90^0 , depending on the polymer content *z*, can be described by means of a square function:

$$R_{90z} = 0.001z^2 + 0.02z + 4.48 \tag{19}$$

The value of the A coefficient was determined using the LSM, and the values of linear correlation coefficients r were calculated. The values of the coefficients are given in Table 10.

Table no 10: Values of directional coefficients A for a straight line and correlation r for natural and modified wood.

coefficient value	Composite				
	K0.0	K 0.35	K 0.43	K 0.48	K 0.56
A	0.7628	0.9193	0.9859	1.0341	1.1071
r	0.9072	0.9480	0.9511	0.9684	0.9793
A_k	0.7470	0.9535	1.0007	1.0302	1.0774
r_k	0.9073	0.9652	0.9635	0.9776	0.9855

The critical value *r* for significance level of 0,05 and 4 degrees of freedom is 0.811. According to data in Table 10, linear correlation increases as the polymer content grows (*r* changes from 0.9071 to 0.9793).

Considering the content of polymer in wood z, the A coefficient can be formulated in the linear form:

$$A_k = A_0 + bz\,. \tag{20}$$

The values of A_0 and *b* coefficients were determined using the LSM. The values calculated on the basis of dependence (20) are given in Table 10. The linear equation describing the A_k coefficient dependent on the polymer content in wood *z* has the following form:

$$A_k = 0{,}747 + 0{,}0059015z\,, \tag{21}$$

furthermore, the values of correlation coefficients r_k were calculated, and the results of calculations were presented in Table 10. The linear equation expressing the dependence of composite strength on the polymer content *z* and angle *α* can be formulated as:

$$R_\alpha = 0.001z^2 + 0.02z + 4.48 + (0{,}75 + 0{,}006z)(90 - \alpha) \tag{22}$$

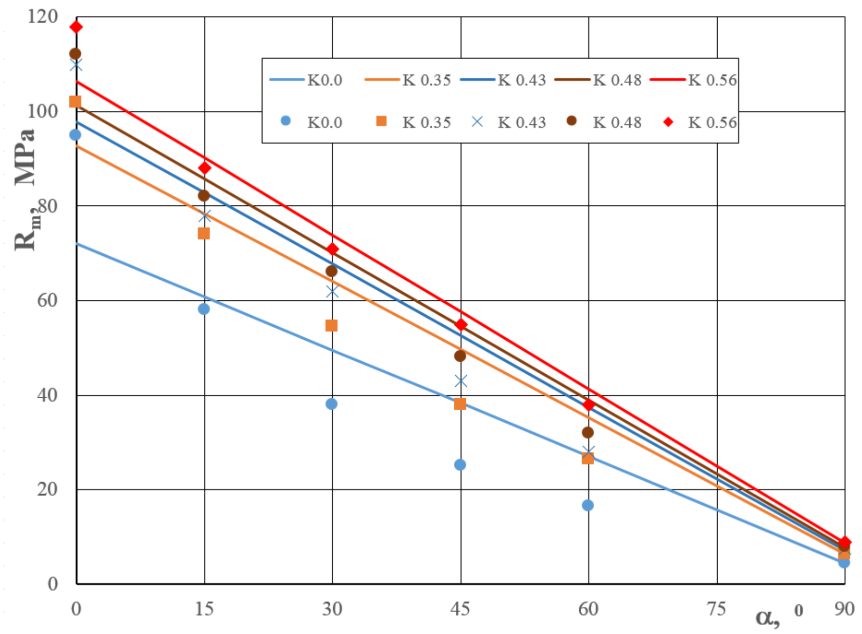

Fig. 7: The results of tensile tests of wood composite K.0÷K0.56: points - experimental data, lines - according to dependEence (22)

Fig. 7 presents the average results of experimental studies and theoretical curves obtained from dependence (22). The course of straight lines describing the strength of composite depending on the polymer content *z* and angle α indicates that straight line functions describe experimental data well, but experimental data should be described in larger detail nonetheless.

Strength description R=R(α) applying the square function:

After a series of tests, parabola 2° crossing empirical points for $\alpha=0^0$, 45^0 and 90^0 was adopted to describe composite strength depending on the composite content *z* and angle α (similarly to the von Mises and Aśkenazi criteria).

The form of the square function:

$$R_\alpha = a + b\alpha + c\alpha^2 \tag{23}$$

Applying the LSM, the a, b, c coefficients were described using dependence (23). For instance, for material K0.0, the form of the square function describing strength depending on angle α was presented as:

$$\hat{R}_\alpha = 91.159 - 2.0590\alpha + 0{,}0123968\alpha^2 \tag{24}$$

Table no 11: The values of a, b, c coefficients of the square function (23) and the correlation coefficient r for modified wood K.0÷K0.56.

Coefficient value	Composite				
	K0.0	K 0.35	K 0.43	K 0.48	K 0.56
a	91.159	100.385	107.762	109.806	115.209
b	-2.059	-1.734	-1.803	-1.667	-1.589
c	0.0124	0.0077	0.0077	0.006	0.0046
r	0.9936	0.9989	0.9979	0.9982	0.9977

The critical value r for significance level of 0.05 and two degrees of freedom is 0.950. According to data in Table 11, the linear correlation coefficient is very high and its value reaches $r > 0.99$.

The calculated a, b, c coefficient values were analyzed against polymer content z. The values of these coefficients are the following:

$a = 89.752 + 0.4152z$ for $r = 0.9677$

$b = -2.0601 + 0.0079z$ for $r = 0.9609$

$c = 0.0125 - 0.00013z$ for $r = -0.9842$.

The square equation expressing the dependence of wood strength on the polymer content z and angle α can be formulated as:

$$R_\alpha = 89.75 + 0.415z + (-2.06 + 0.008z)\alpha + (0.0125 - 0.00013z)\alpha^2 \quad (25)$$

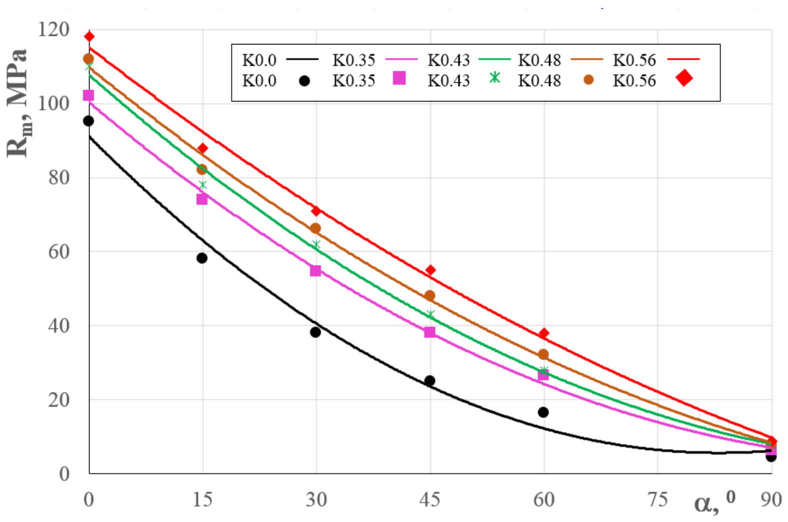

Fig. 8: Tensile strength: points - experimental data, curves - according to dependence (25)

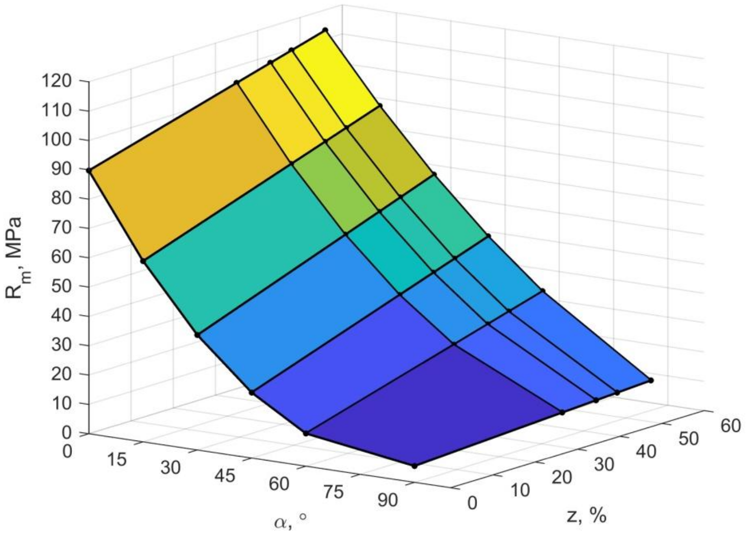

Fig. 9: A plane diagram illustrating the dependence of tensile strength of modified wood on polymer content z and angle α (based on dependence 25)

Figures 8 and 9 presents the averaged results of experimental studies and theoretical curves described by means of dependence (25). The course of this function indicates that the square function is definitely the best method to describe experimental data. The empirical dependences applied for wood which was superficially modified with methyl methacrylate have been found to be more accurate in describing the strength properties of wood composites, compared to the strength criteria proposed by von Mises and Aśkenazi.

III. Analysis of test results

According to the experimental data obtained, the strength properties of superficially modified wood increase as the polymer content grows. The highest values of tensile strength $\overline{R}_m$ =118 MPa were recorded for composite of PMA content 0.56kg/kg (K0.56) when tension is applied along the fibers $\alpha=0^0$. The lowest value of tensile strength was recorded for natural wood (K0.0) stretched across the fibers ($\alpha=90^0$).

The analysis of experimental results applied to the impact of PMA content and the direction of the load applied relative to the direction of the fibers (angle α). An increase in polymer content caused the structure to become more homogenized, which in turn improved the specimen's strength properties. And although the increase in polymer content caused the strength of the material to increase more than twice (R_m increased from 4.5MPa to 9 MPa) for the crosswise fiber layout ($\alpha=90^0$), the strength of this material still remained low for crosswise load.

The strength of composite decreased as the angle α increased. This particularly applied to natural wood. It was further observed that crosswise modified wood samples were clearly more strengthened as the polymer content increased. This strengthening increased proportionally to the impregnation level.

Known strength criteria applicable to anisotropic materials (von Mises, Aśkenazi) as well as the author's own criteria: linear and square were used to describe the strength of superficially modified wood. Each of the 4 diagrams presented in Figures 5 ÷ 8 was drawn up for 5 sets of samples characterized by different PMA contents and angles α .

The results of experimental studies applying the von Mises and Aśkenazi criteria, used to describe material strength, proved to be insufficiently accurate. These criteria are inadequate for describing the strength of superficially modified wood.

The linear and square functions proposed describe the strength properties of composite with a much higher accuracy. This is proven by the correlation coefficient which is 0.90 to 0.98 for the linear function and above 0.99 for the square function.

IV. Conclusions

The study was conducted to assess the strength properties of wood which was superficially modified with PMM. The strength properties of wood composite were analyzed against the amount of polymer contained in wood and angle α of deflection of the fibers with reference to the direction of the load. The study has indicated that increasing the polymer content strengthens the modified wood structure, and thus improves the strength properties in all load directions relative to the fiber orientation. Composite with the highest polymer content, stretched along the fibers, was found to have the highest tensile strength. Therefore, modified wood structures subject to the highest loads should be designed to be loaded along the fibers. The best results were obtained for structures made of modified wood which were bent. Superficial modification of wood strengthens its external layers which are reinforced through modification and are then responsible for carrying the load.

The strength properties of wood impregnated with PMM were described according to known strength criteria applicable to anisotropic materials, i.e. the von Mises yield criterion and the Aśkenazi criterion. The application of these criteria in describing the strength properties of modified wood indicated that these criteria insufficiently describe the strength properties. Furthermore, the forms of these criteria do not take into account the degree of impregnation of superficially modified wood.

The proposed method of description of strength of impregnated wood, applying the linear and square functions, was found to be more accurate with respect to experimental results. For practical reasons, a dependence (25) can be proposed in order to describe experimental data with the highest possible accuracy.

Studies of superficially modified wood have indicated that new solutions and new models should be pursued in order to provide a better description of experimental data obtained through testing of unknown materials. This particularly applies to composite materials.

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