

Modeling and Simulating Radioactivity in the Physics Laboratory of Under-Graduate Students

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Abstract: An experiment to simulate radioactive decay could be easily performed with a set of 120 dices and random-number generator in the laboratory environment. The nuclei are imagined to be indexed (1 to 6 and 1 to 120) to demonstrate the role of decay constant. The appearance of the index number of particular nuclei is interpreted as a signal that these have disintegrated. The number of surviving nuclei is an exponential function of the number of draws, and represents the radioactive decay law, with small fluctuations. This procedure is a precise realization of the conventional probabilistic derivation of the radioactive decay law. The same problem is also modeled and simulated using concepts of Spreadsheets, VBA and random numbers to enhance the understanding of students.

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I. Introduction

In 1896, A.H. Becquerel accidentally discovered radioactivity. He was studying the fluorescence and phosphorescence of compounds irradiated with visible light. Thus radioactive decay is the name given to the natural process by which an atomic nucleus spontaneously transforms itself into another nucleus with the emission of alpha, beta or gamma ray. If a sample consists of N_0 radioactive nuclei at time $t = 0$, and if $N(t)$ denotes the number of the nuclei remaining at any later time t , then the function $N(t)$ decreases exponentially with time, as given by equation (1).

$$N(t) = N_0 e^{-\lambda t} \quad (1)$$

where λ is called decay constant.

$\Delta N = -\lambda N(t)dt$ is the number of nuclei decayed in time ' dt '.

or

$$\lambda = \frac{\Delta N}{N(t)dt} \quad (2), \quad \text{negative sign is ignored.}$$

Above expression shows that λ is probability of decay per unit time. Let us draw an analogy from the concept of probability. Suppose there are 100 balls in a bag and 25 are blue in color. Then probability of drawing a blue ball is:

$$\text{Probability} = \frac{\text{Number of favourable cases}}{\text{Total number of cases}} = \frac{25}{100} = \frac{\text{Number of favourable cases}}{\text{Total number of cases}}$$

Equation (2) can be re-written as:

$$\lambda = \frac{\Delta N}{N(t)dt} = \frac{\text{Number of favourable cases i.e. number of decayed nuclei}}{\text{Total number of nuclei (dt = 1)}}$$

That is why, λ is called decay probability.

In many cases, students are not used to exponential mathematics and lack the understanding necessary for interpreting equation (1). As the use of radioactive material in the classroom is not always practical or advisable, several alternative activities for modeling radioactive decay have been suggested from time to time.

For example, in Hughes and Zalts' [1] studies, an exponential decay graph was constructed quite easily with using a stiff paper, a scissor, a glue, a measuring tape, a pencil and strips. In addition, Klein and Kagan [2] described in their study a visual and interactive use of dice to develop student understanding of radioactive decay. Jesse [3] described a computer simulation, based on dice game, for explaining radioactive decay.

This study aims teaching of the law of radioactive decay with modeling and simulation.



Figure 1: Set of dices.

II. Methodology

In this study, two approaches are presented. These are:

A. Physical Model

As per lab environment is concerned, 120 pieces of dices are taken to simulate the "Radioactivity" [Dice faces are numbered from 1 to 6 or they have dots corresponding to number representing the face]. We know that when a dice is thrown up (tossed in the air), it will fall only with one face showing up – it could be any face numbered from 1 to 6. That implies the probability of any one event is $1/6$. For example, for a dice to fall with face number 5, its chances are $1/6$. To model the radioactivity, it is decided that if the dice falls with face 5, it is assumed that this represents the decay of atomic nucleus. Therefore the said dice or dices will be taken out from the sample. The left over sample i.e. dices will be again tossed and the process is repeated. Every throw or toss is considered as a unit of time. The graph is plotted between the number of throws (time) and dices left over (atomic nuclei survived). It is found that the plot of decay is exponential in nature. The experiment is performed by a group of students in the laboratory. The data obtained is presented in Table 1.

To verify the authenticity of decay law, the data is procreated using relation

$$N(t) = N_0 e^{-\lambda t}$$

where $\lambda=1/6$ (λ is probability) and $N_0=120$.

It is found that both the numbers (modeled and procreated) are almost same within certain inherent errors (see Table 1). This represents the success of modeling the phenomenon of radioactivity in the lab environment.

B. Computer Based Model - Simulation

Computer simulation is a powerful methodology for design and analysis. The overall approach in computer simulation is to represent the dynamic characteristics of a real world system in a computer model. Since most dynamic problems in practice cannot be represented and solved fully using mathematical equations, computer simulation is a powerful and flexible methodology in complex systems analysis. In this case, we make use of VBA (Visual Basic for Applications) to bring the radioactivity into the Physics Laboratory with the help of computer and spreadsheets (MS-EXCEL). The students and teachers are required to have little knowledge of the objects used in simulation and modeling i.e. VBA, Spreadsheets and random number generator. Therefore, computer simulation can be used by the students to compare the real experiment and simulation. The occurrence of desired number by random number generator is linked to the decay of atomic nucleus. The process consists of following steps:

- i. Let there are 120 atomic nuclei (120 dices) in the beginning.
- ii. Invoke the random number generator 120 times. It is equivalent to tossing 120 dices simultaneously.
- iii. Observe the frequency of desired event (any one random number between 1 and 6, say 5). Its occurrence means decay of atomic nucleus. Let it be 'f'.
- iv. Now the nuclei survived after first toss are:
N=120- f.
- v. Repeat the above steps till all the nuclei have decayed.
- vi. Plot a graph between number of survived nuclei and number of tosses (one toss is equivalent to one unit of time).

The screen shot of this simulation is shown in Figure 2.

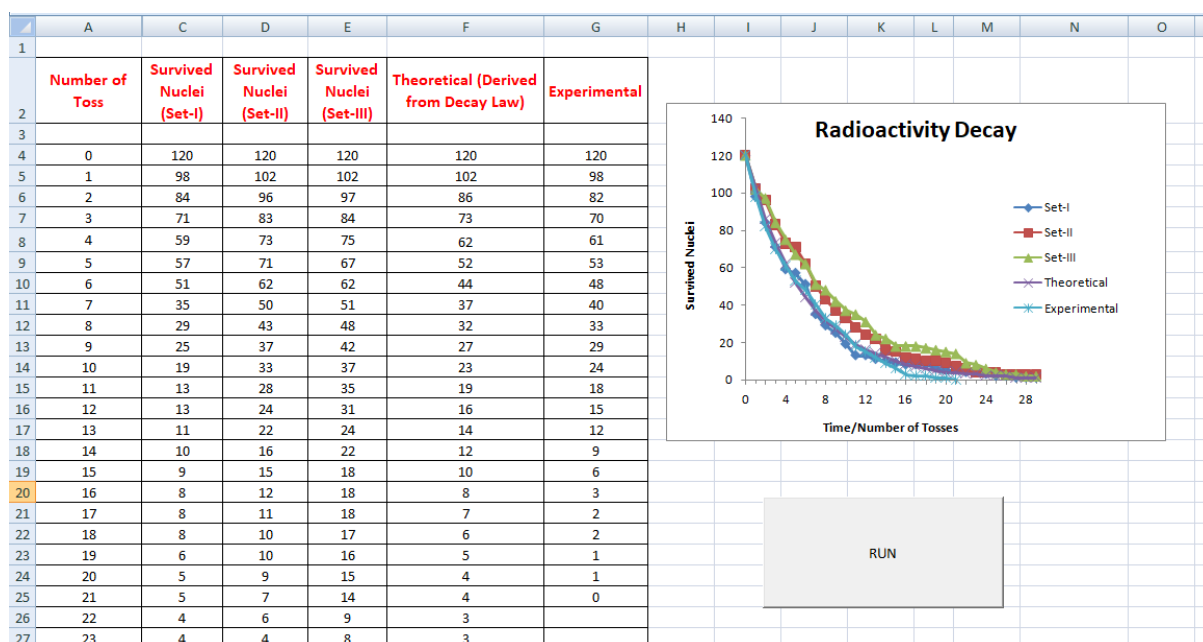


Figure 2: Screen shot of simulation of radioactivity.

VBA Code

The following code is used to procreate the random numbers and fill the spreadsheet with the data:
Private Sub CommandButton1_Click()

Dim MyValue As Double, j As Integer, NI As Integer

NI = 120 'Number of dices

Cells(4, 14) = NI

pp = 1

For j = 1 To 100 'j=Number of tosses

k = 0

For kk = 1 To NI ' This loop means single toss

MyValue = Int((7 * Rnd) + 1) ' Generates number between 1 and 6

If MyValue = 5 Then

k = k + 1

End If

Next kk

Cells(pp, 2) = k

Cells(pp, 1) = j

Cells(pp, 3) = NI - k ' Remove the decayed nuclei

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If (N1 - k) <= 0 Then
GoTo 100
Else
N1 = Cells(pp, 3)
pp = pp + 1
End If
Next j

100 End
End Sub
    
```

The above experiment is simulated in the physics laboratory. The data obtained is presented in the Table 1.

Number of Toss (t)	Procreated Numbers by Random Number Generator (SIMULATION)			Theoretical (Derived from Decay Law $N = 120e^{-\frac{1}{6}t}$)	Experimental (MODELING)
	Survived Nuclei (Set-I)	Survived Nuclei (Set-II)	Survived Nuclei (Set-III)		
0	120	120	120	120	120
1	103	102	102	102	98
2	84	96	97	86	82
3	76	83	84	73	70
4	61	73	75	62	61
5	45	71	67	52	53
6	40	62	62	44	48
7	36	50	51	37	40
8	32	43	48	32	33
9	30	37	42	27	29
10	27	33	37	23	24
11	22	28	35	19	18
12	18	24	31	16	15
13	16	22	24	14	12
14	16	16	22	12	9
15	13	15	18	10	6
16	11	12	18	8	3
17	10	11	18	7	2
18	9	10	17	6	2
19	8	10	16	5	1
20	7	9	15	4	1
21	7	7	14	4	0
22	6	6	9	3	
23	4	4	8	3	
24	4	4	6	2	
25	4	4	4	2	
26	4	3	3	2	
27	3	3	3	1	
28	3	3	2	1	
29	3	3	2	1	

Table 1: Theoretical, Experimental and Simulated data obtained during activity

III. Results and Discussion

Before attempting this model in laboratory, it is presumed that the students have idea of radioactivity. However, to them the nature of shape of exponential decay was somewhat mystery. May be they were not good at visualizing the shape from mathematical equations. The experimental, procreated data using theoretical model and simulated data are plotted and shown in Figure 3. All of these plots obey the exponential rule – the law of radioactivity. But plotting graph from the experimental data (Physically Modeled) was an attempt to enable them to understand - exponential function and radioactivity. At least by having hands on experience, students could develop familiarity with the law of radioactivity. Even their response after completion of this activity was fairly good. At least they were now in position to understand and explain that what happens in radioactivity and how it is expressed in mathematical form i.e. exponential rule. At the same time computer

simulation also enabled them to comprehend the probabilistic nature of radioactive decay. The dynamic behavior of the simulated phenomenon could be seen from the fluctuating graphs for every run – due to different numbers generated.

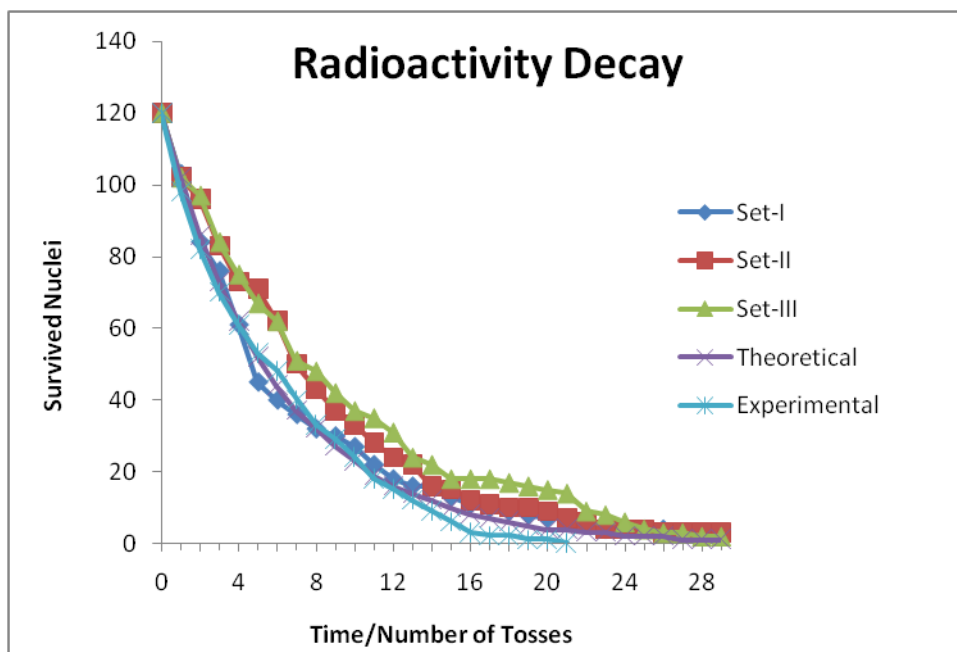


Figure 3: Experimental, Theoretical and Procreated (Simulated) radioactive decay.

To understand the physical significance of the decay constant, the above model was slightly varied. Now the 120 dices were numbered from 1 to 120 and placed in a bowl. The random number between 1 and 120 is generated (This is equivalent to toss a dice with 120 faces). The initial appearance of number generated is interpreted as decay of nucleus. The dice with that number is taken out of bowl. If the same number appears again, this does not change the population of the nuclei (dices), although it increases the tosses i.e. attempts to generate random number. The data is plotted in Figure 4 to draw the comparison. The activity does enable the students to understand the role of decay constant in radioactivity and they were able to speak about consequences of small or large decay constant.

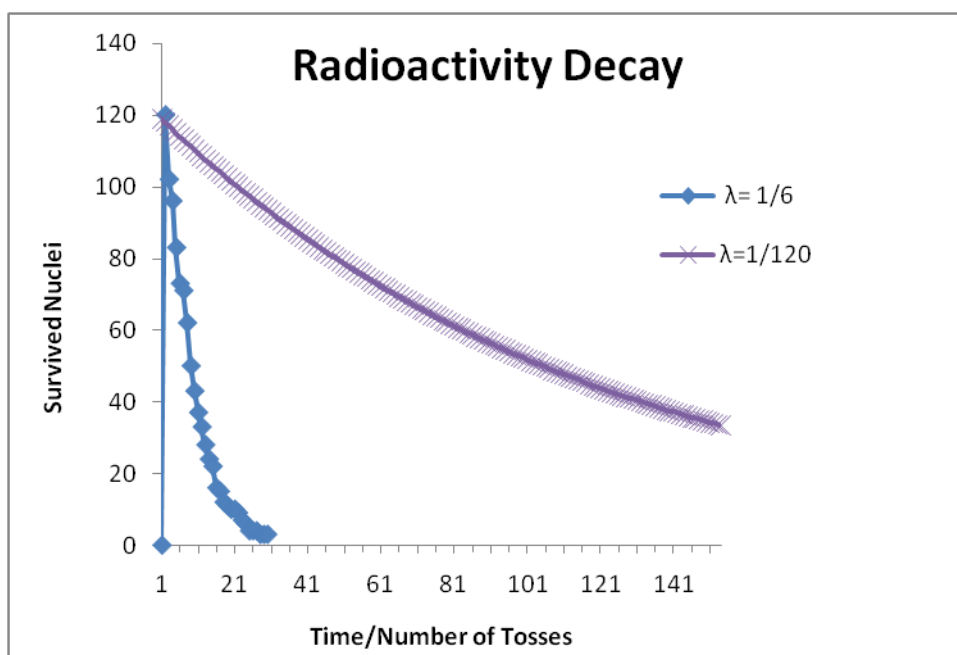


Figure 4: Significance of decay constant, λ .

Another important point about radioactivity is “Half Life”. To explain practically that half life is independent of strength of sample (Number of dices); the simulation was done with different number of dices. The half life was computed for each simulation. The result was almost same within statistical fluctuations. This observation could have striking effect on the belief of students regarding physical parameter that the information about it is not just a statement recorded in the book but can be realized through proper modeling and simulation techniques.

IV. Conclusions

The students were enabled to better understand the subject by verifying the law of radioactive decay and understanding the role of decay constant with small variation in the physical model. By varying the number of dices, it could easily be demonstrated that the radioactivity (rate constant) and “Half Life” are independent of the number of dices. The activity also enabled the students to know about half life concept and its determination thus strengthening their logical understanding of the subject.

Acknowledgement

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References

- [1]. Hughes E.A. & Zalts, A. Radioactivity in the Classroom, *Journal of Chemical Education*, 77, 613-614 (2000).
- [2]. Klein, L. & Kagan, D. “Radio-Active” Learning: Visual Representation of Radioactive Decay Using Dice, *The Physics Teacher*, 48, 45 (2010).
- [3]. Jesse, K.E., Computer Simulation of Radioactive Decay, *The Physics Teacher*, 41, 542-543 (2003).

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