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Ramond-Ramond charge calculation for D brane in B field

TetianaObikhod, Ievgenii Petrenko[1]
[1]*Kyiv Institute for Nuclear Research NAS of Ukraine*

Abstract: *The paper is connected with searches for the Ramond-Ramond charge of D branes in the presence of B field. The consideration of B field inclusion is an important physical and mathematical unsolved problem, which is connected with K group calculations of twisted bundles. Considered two cases of vector bundles, Azumaya and Rosenberg algebras and analyzed their K group realization.*
Key Word: *D branes; B field; Azumaya and Rosenberg algebras; twisted bundles; Ramond-Ramond charge.*

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I. Introduction

One of the most interesting question of Ramond-Ramond (RR)-charge classification of D-brane is the description of corresponding vector bundles characterized by Dixmier—Douady invariant by twisted K-theory group [1]. As is known [2], RR fields on D-branes are sources for fields of type II string theory. As quantum RR fields are classified by twisted K-theory, we'll present the mathematical consideration of RR-charge in terms of C*-algebra on Hilbert space and corresponding topological invariant, element of twisted K-group.

Historically, RR-charge appeared in the type II closed superstring theory with gauge fields from RR sectors of string Hilbert space, [3]. Inclusion of boundary conditions on open string endpoints leads to hyperplane, the D-brane, with p spatial and one timelike dimension. The quantum charge calculation of D-brane includes the exchange of open and closed string between the two D-branes. From minimum quantum, n = 1, it was argued that D branes are RR-charged objects. The the exchange by these charges between D-branes is carried out by strings, just as the excitation in an atom is removed by electronic transitions between different levels. So, the classification of D-branes acquires a new mathematical interpretation, which will be presented in this paper.

II. K group calculations

The problem of D-brane charge classification was raised by Witten, [4]. He showed that analyzing of the brane-antibrane system lead to the identification of D-brane charge as an element of the K-theory of the spacetime manifold X as a base for some vector bundles corresponding to D branes. Thus, the interpretation of D-brane charges in terms of K-theory is connected with the basic reason that D-branes carry vector bundles. For IIB string theory one considers a configuration of equal number of D9 and anti-D9-branes carrying vector bundles E and F. The pair (E, F) defines a class in K-theory.

From [5] is known, that wrapped D-branes around supersymmetric cycles f: W → S with vector-bundle E → W, called the Chan-Paton bundle are charged under the RR gauge fields. RR charge of D brane is determined by formula

$$Q = \mathrm{ch}(f_! E)\sqrt{\hat{A}(\mathcal{T}S)},(1)$$

where $\mathcal{T}$S is the tangent bundle to spacetime and $f_!$ is the K-theoretic Gysin map.

But the question is connected with finding of RR charge of D branes in topologically nontrivial B fields. In the presence of the Neveu-Schwarz B-field interacting with D brane the field strength, H, is determined by formula

$$H_{\mu\nu\rho} = \partial B_{\nu\rho} + \partial B_{\rho\mu} + \partial B_{\mu\nu} \qquad (2)$$

From the paper [1] is well known, that the incorporation of Neveu-Schwarz B-field with three-form field strength H and characteristic class $[H] \in H^3(X, \mathbb{Z})$allows to interpret the gauge fields on the D-brane as connections over noncommutative algebras rather than as connections on vector bundles, [6]. As the cancelation of global string worldsheet anomalies requires $[H]$ to be a torsion element, the incorporation of nontorsion $[H]$ leads to the limit n →∞ of principal $PU(H) = U(H)/U(1)$bundles over X with H - an infinite dimensional, separable, Hilbert space. For such bundle sections became C*-algebra of continuous sections of the algebra bundle over infinite dimensional, separable, Hilbert space and C* algebra is itself became Hilbert A-module.

There must be the modifications in consideration of the sections of bundles corresponding to such D branes, [7]

$$SU(n)/Z_n \longrightarrow P_H \downarrow X$$

(3)

$$\lim_{n\to\infty} SU(n)/Z_n \longrightarrow P_H \downarrow X$$

Isomorphism classes of principal PU(H) bundles over X are parametrized by $H^3(X, \mathbb{Z})$. The upper part of (3) is principal bundle called Azumaya bundle with n[H] = 0, where $H_{\mu\nu\lambda} = 0$, $B_{\mu\nu} \neq 0$; the lower part of (3) is principal bundle with $[H] \neq 0$, where $H_{\mu\nu\lambda} \neq 0$, $B_{\mu\nu} \neq 0$ called Rosenberg bundle [8].

Vector bundles associated with principal one are the following

$$E_H = P_H \times M_c(C), \quad where \quad Aut(M_c(C)) = SU(n)/Z_n \qquad (4)$$

$$E_H = P_H \times K, \quad where \quad Aut(K) = \lim_{n\to\infty} SU(n)/Z_n \qquad (5)$$

where $M_c(C)$ is n × n matrix algebra,$\mathcal{K}$is the algebra of compact operators. It turns out that isomorphism classes of locally trivial bundle $\varepsilon_{[H]}$ over X with fiber $\mathcal{K}$ and structure group AutType equation here.($\mathcal{K}$) are also parametrized by the cohomology class in $H^3(X, \mathbb{Z})$ called the Dixmier-Douady invariant of $\varepsilon_{[H]}$and denoted by $\delta(\varepsilon_{[H]}) = [H], [H] \in H^3(X, Z)$, [9]

As was stressed by Witten in [10], for two Azumaya bundles, W, with string between these twisted bundles, the algebra of W - W open string field theory reduces to the algebra $A_{W(X)}$ of linear transformations of bundle W. In general, W is locally trivial, so $A_{W(X)}$is isomorphic toA(X) $\otimes M_N$, where M_N is the algebra of N × Ncomplex-valued matrices. There is also used the fact that for distinct twisted bundles W and W′, the corresponding algebras are "Morita-equivalent" and $K(A_W) = K(A_{W'})$. There K-theory is taken in [H] = 0 case for the noncommutative Azumaya algebra over compact space X. As was stressed in [4] in the case of Azumaya bundles the groups K(X) and K(X, [H]) over compact space X are rationally equivalent.

In most physical applications, [10] for the case of Type IIB string theory with nontorsion $[H] \neq 0$ we have infinite set of D9 or anti D9 branes with infinite rank twisted gauge bundle E or F. D brane charge is classified by K_H group of pairs (E,F) modulo the equivalence relation.

So, we can say, that gauge fields on D brane in the presence of B field are interpreted as connections over noncommutative algebras, [1]. Thus, D-brane charges in the presence of B field with nontrivial [H] are classified by K-theory of some noncommutative algebra, C*-algebra of continuous sections of isomorphic classes of locally trivial bundles ε[H] over X with fibre$\mathcal{K}$ and structure group PU(H) = Aut($\mathcal{K}$).

$$K^j(X,[H]) = K_j(C_0(X, \varepsilon_{[H]})), \quad j = 0,1. \qquad (6)$$

K is the C -algebra of compact operators on H-an infinite dimensional, separable, Hilbert space. Therefore, D-brane charges in the presence of a B-field are identified with defined by Rosenberg twisted K-theory of infinite-dimensional, locally trivial, algebra bundles of compact operators, introduced by Dixmierand-Douady.

The set of all linear operators form a linear space. In particular:

- the sum of the linear operators and the product of the linear operator by number are determined;
- the norm of the operator is defined;
- triangle inequalities are satisfied;
- the validity of the homogeneity property of the norm is verified.

Let X, Y be linear normalized operators. A linear operator A: X → Y is said to be bounded if there is a M = const such that

$$| Ax | \leq M \, | x | \quad \text{for} \quad \text{any } x \in X.$$

A classic example of a C* -algebra is the algebra B(H) of bounded (or equivalent continuous) linear operators defined on a complex Hilbert space H.

The classification of algebras with locally compact spectrum X is facilitated by stable isomorphism classes of algebras (for example, A and B are isomorphic, if A ⊗K ≃B ⊗K) over locally compact Hausdorff space with countable basis of open sets. The reason is connected with the fact that the bundle $\varepsilon_{[H]}$ is the unique locally trivial bundle over X with$\delta(\varepsilon_{[H]}) = [H]$. There is bijection between isomorphism classes of algebras whose irreducible representations are infinite-dimensional, "locally trivial" and Cech cohomology group $H^3(X, \mathbb{Z})$.

To compute K(A) we can use Mayer-Vietoris sequence [8], from which the Dixmier - Douady invariant is determined as the image in Cech cohomology.

$$H^2(PU(H), Z) \to H^3(X, Z) \to 0. \qquad (7)$$

Here $X = Y \cup Z$, Y and Z are closed subsets of X and $Y \cap Z \to PU(H)$. The interesting case is connected with $\{Y_n\}$ - some covering of X and algebra A restricts to $C(Y_n) \otimes K$ on Y_n, $n \to \infty$, and $PU(H)$ is classifying space of line bundles determined for intersections of Y_n. Thus for case, $[H] \neq 0$, $K_0(A) = 0$ and $K_1(A) \cong \mathbb{Z}_n$.

Thsame result can be obtained in another way. From [11] it is known, that it can be determined an extension Ext(A,C) of C* algebras A and C by B algebra together with isomorphisms and for which the following sequence is exact

$$E: 0 \to A \xrightarrow{\alpha} B \xrightarrow{\beta} C \to 0 \qquad (8)$$

From long exact sequence of abelian groups of C* algebras

$$\ldots \to Ext_0(A) \to Ext(C) \to Ext(B) \to Ext(A) \to \ldots,$$

can be received the following result $Ext_0(A) \cong \mathbb{Z}_n$.

III. Conclusions

The main purpose af the paper is connected with searches for the RR charge of D branes in the presence of B field. The case of usual D brane RR charge is studied and the answer is known. The presence of the B field is an important physical and mathematical unsolved problem.

Our task boils down into two issues. First, there is considered [H] = 0 case for the noncommutative Azumaya algebra over compact space X. The algebra of W - W open string field theory between these twisted Azumaya bundles reduces to the algebra $A_{W(X)}$ of linear transformations of the bundle W. We also used the fact of "Morita-equivalence" of distinct twisted bundles W and W′ and rationally equivalence of the groups K(X) and K(X, [H]) over compact space X.

The second case is more important, less studied and connected with Rosenberg bundles and with the need to calculate the corresponding K-group. D-brane charges in the presence of B field with nontrivial [H] are classified by K-theory of some noncommutative algebra, C^*-algebra of continuous sections of isomorphic classes of locally trivial bundles. But the description for torsion elements is more natural as the bundle $\varepsilon_{[H]}$ is the unique locally trivial bundle over X. We have considered only compact space X and calculated $Ext_0(A) \cong \mathbb{Z}_n$.

These results can be compared with the corresponding calculations of massless Ramond states of open strings connecting D-branes wrapped on submanifolds of Calabi-Yau's, with holomorphic gauge bundles. The massless Ramond spectra of open strings connecting D-branes are counted by Ext groups and obtained results could be reinterpreted in the language of particles for the corresponding RR charges. The realization of module space in terms of SU(5) multiplets gives supersymmetric matter content [12]. So, it would be interesting to understand the particle realization for the considered twisted bundles.

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