IOSR Journal of Applied Physics (IOSR-JAP)
e-ISSN: 2278-4861.Volume 11, Issue 3 Ser. II (May. – June. 2019), PP 25-29
www.iosrjournals.org

The Impact of the Electric susceptibility on Electron Effective Mass

Elmardi.A. M. Ali[1], M. H. M. Hilo[2], KH. M. Haroun[3], Lutfi Mohammed Abdalgadir[4], Mubarak Dirar Abdallah[5]

[1]*(Department of Physics, College of Science and Arts-Alrass/Qassim University, Saudi Arabia)*
[2,5]*(Department ofPhysics, College of Science / Sudan University of Science & Technology, Sudan)*
[3]*(Department ofPhysics, College of Radiobiological and Imaging science / Al Zaiem Al Azhari University, Sudan*
[4]*(Department ofPhysics, College of of Science & Humanities-Hurymilla / ShaqraUniversity, Saudi Arabia)*
Corresponding Author:- Elmardi.A. M. Ali

Abstract: *The electron effective mass directly depends on the electric field where the electron presented in it. In this study the electron effective mass was studied by derivation (mathematically) for the equations of motion(velocity and acceleration).Considering that the electron presented in an electric field, an equation for electron effective mass was found. When neglecting the electric field, the effective mass is equal to the ordinary mass.*

Date of Submission: 13-05-2019 Date of acceptance: 30-05-2019

I. Introduction

In solid state physics, a particle's effective mass (often denoted m*) is the mass that it seems to have when responding to forces[1], or the mass that it seems to have when interacting with other identical particles in a thermal distribution. One of the results from the band theory of solids is that the movement of particles in a periodic potential[2], over long distances larger than the lattice spacing, can be very different from their motion in a vacuum. The effective mass is a quantity that is used to simplify band structures by modeling the behavior of a free particle with that mass [3]. For some purposes and some materials, the effective mass can be considered to be a simple constant of a material [4]. In general, however, the value of effective mass depends on the purpose for which it is used, and can vary depending on a number of factors. For electrons or electron holes in a solid, the effective mass is usually stated in units of the rest mass of an electron, me (9.11×10^{-31} kg)[5]. In these units it is usually in the range 0.01 to 10, but can also be lower or higher-for example, reaching 1,000 in exotic heavy fermion materials, or anywhere from zero to infinity (depending on definition) in graphene. As it simplifies the more general band theory [6], the electronic effective mass can be seen as an important basic parameter that influences measurable properties of a solid, including everything from the efficiency of a solar cell to the speed of an integrated circuit [7].

II. The Effective Quantum Model

In this model the electrons or particles are need to be free. Thus the energy expression can be given by [8]:

$$E = \frac{\hbar^2 k^2}{2m^*} = \frac{\hbar^2 {k^*}^2}{2m} \quad (1)$$

The effect of the field is embedded in the mass for the second term, while it is embedded in the wave number in the third term. Thus

$$\frac{m^*}{m} = \left(\frac{k}{k^*}\right)^2 \quad (2)$$

The wave number is given for vacuum by:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{\lambda f} = \frac{\omega}{c} \quad (3)$$

While for any medium it is given by:

$$k^* = \frac{2\pi}{\lambda} = \frac{2\pi f}{\lambda f} = \frac{\omega}{v} = \omega\sqrt{\mu_o \varepsilon_o \mu_r \varepsilon_r}$$

$$k^* = \sqrt{\mu_o(1+\chi_m)\varepsilon_o(1+\chi_e)} \quad (4)$$

Where χ_m is magnetic susceptibility and χ_e is electric susceptibility
Where the magnetic and electric susceptibility are given by [9]:

$$\varepsilon = \varepsilon_o(1+\chi_e)\mu_o(1+\chi_m)(5)$$

For any medium one gets [10]

$$k^* = \omega\sqrt{\mu_o\varepsilon_o(1+\chi_m)(1+\chi_e)}$$

$$k^* = \frac{\omega}{C}\sqrt{(1+\chi_m)(1+\chi_e)} \qquad (6)$$

In view of equation (2) and equation (6) the effective mass is given by:

$$m^* = m\left(\frac{1}{(1+\chi_m)(1+\chi_e)}\right) \qquad (7)$$

III. Elections for Free Vibration Model Atoms

In this model, one ignores the contribution of magnetic moment, $\chi_m = 0$ thus equation (7) becomes as

$$m^* = m\left(\frac{1}{(1+\chi_m)}\right)$$

Consider a free vibrating electron or electric dipoles to discuss the effective mass due to the change of electric susceptibility only. The equation of motion becomes [11]

$$m\ddot{x} = -k_o x + eE = -m\omega_o^2 x + eE(8)$$

$$m\dot{v} = -k_o x + eE = -m\omega_o^2 x + eE \qquad (9)$$

For equation (8) let $x = x_o e^{-i\omega t}$, so the velocity and acceleration can be found as

$\dot{x} = -i\omega x \quad , \ \ddot{x} = -\omega^2 x$

Substituting these results in equation (8), one gets

$$-\omega^2 m x = -m\omega_o^2 x + eE(10)$$

$$x = \frac{eE}{m(\omega_o^2-\omega^2)}(11)$$

The electric moment is given as[12]:

$$p = enx = \chi_e E$$

Using equation (11), one gets

$$p = \frac{e^2 nE}{m(\omega_o^2-\omega^2)} = \chi_e E(12)$$

$$\chi_e = \frac{e^2 n}{m(\omega_o^2-\omega^2)}(13)$$

Inserting equation(13) into equation(7)for[$\chi_m = 0$] one gets

$$m^* = \frac{m}{1+\chi_e} = \frac{m}{1+(e^2 n/m(\omega_o^2-\omega^2))}$$

$$m^* = \frac{m^2(\omega_o^2-\omega^2)}{[m(\omega_o^2-\omega^2)+e^2 n]}(14)$$

For Equation (10), let $v = v_o e^{-i\omega t}$, so the displacement and acceleration can be found as

$$x = \int v dt = \frac{v_o e^{-i\omega t}}{-i\omega}$$

$$= \frac{-v}{i\omega} = \frac{i^2 v}{i\omega} = \frac{iv}{\omega}$$

$$a = \dot{v} = -i\omega v_o e^{-i\omega t} = -i\omega v(15)$$

Substituting equation(15) into equation (10)one gets

$$-im\omega v = \frac{-m\omega_o^2(iv)}{\omega} + eE$$

$$im(\omega_o^2-\omega^2)v = e\omega E$$

$$v = \frac{e\omega E}{im(\omega_o^2-\omega^2)}(16)$$

Using the definition of electric density one gets

$$\square = \frac{\partial p}{\partial t} = \frac{\partial \chi_e E e^{-i\omega t}}{\partial t} = -i\omega\chi_e E e^{-i\omega t}$$

$$\square = -i\omega\chi_e E \qquad (17)$$

On the other hand $\square = nev = \frac{ne^2\omega}{im(\omega_o^2-\omega^2)}E$

Equating this result to equation (17) one gets

$$\chi_e = \frac{ne^2}{-i^2 m(\omega_o^2-\omega^2)} = \frac{ne^2}{m(\omega_o^2-\omega^2)}(18)$$

Using equation*s* (13) and (18) one gets:

$$m^* = \frac{m}{1+\chi_e} = \frac{m^2(\omega_o^2-\omega^2)}{[m(\omega_o^2-\omega^2)+e^2n]} \quad (19)$$

This is quite obvious since when no polarized atom exists and no field applies to the electrons, they will be become as in a vacuum.

IV. Electric Effective Mass for A Two Electrons in Frictional Medium Electric

Considering the friction term, the equations of motion can be written as

$$m\ddot{x} = eE - \gamma v \quad (20)$$

$$m\dot{v} = eE - \gamma v \quad (21)$$

Using displacement approach in equation (20), one can assume

$$x = xe^{-i\omega t}$$

$$v = \dot{x} = -i\omega x$$

$$\ddot{x} = i^2\omega^2 x = -\omega^2 x \quad (22)$$

$$-m\omega^2 x = eE - i\gamma\omega x$$

$$(i\gamma\omega - m\omega^2)x = eE$$

$$x = \frac{eE}{(-m\omega^2 + i\gamma\omega)}$$

Multiplying and dividing by $(-m\omega^2 - i\gamma\omega)$, gives

$$x = \frac{e(-m\omega^2 - i\gamma\omega)E}{(-m\omega^2 - i\gamma\omega)(-m\omega^2 + i\gamma\omega)}$$

$$x = \frac{e(-m\omega^2 - i\gamma\omega)E}{(m^2\omega^4 + \gamma^2\omega^2)} \quad (23)$$

Using the definition of electric moment one gets the electric susceptibility as

$$\chi_e = \frac{-ne^2(m\omega^2 + i\gamma\omega)}{(m^2\omega^4 + \gamma^2\omega^2)} \quad (24)$$

Substituting equation (24) into equation (19) one gets

$$m^* = \frac{m(m^2\omega^4 + \gamma^2\omega^2)}{-ne^2\omega^2 + i\gamma\omega + m^2\omega^4 + \gamma^2\omega^2} \quad (25)$$

The velocity approach in equation (21) requires to assume

$$v = v_o e^{-i\omega t}$$

$$a = \dot{v} = -i\omega v \quad (26)$$

Inserting equation (26) into equation (21) gives

$$-i\omega m v = eE - \gamma v$$

$$(\gamma - im\omega)v = eE$$

$$v = \frac{e}{\gamma - im\omega}E \quad (27)$$

But the current density is defined by

$$J = nev = \frac{ne^2}{\gamma - im\omega}E \quad (28)$$

$$J = \frac{ne^2E}{\gamma - im\omega}$$

Equating the last expression for J to equation (17) one gets

$$-i\omega\chi_e E = \frac{ne^2E}{\gamma - im\omega}$$

$$\chi_e = -\left[\frac{ne^2}{\omega(i\gamma + m\omega)}\right] \quad (29)$$

Substituting equation (29) into equation (25) one gets

$$m^* = \frac{\omega m(i\gamma + m\omega)}{\omega(i\gamma + m\omega) - ne^2}$$

$$m^* = \frac{im\omega\gamma + m^2\omega^2}{i\omega\gamma + m\omega^2 - ne^2} \quad (30)$$

V. Free Vibrating Elections In Frictional Medium Electric Effective Mass

$$m\ddot{x} = eE - k_o x - \gamma v$$

$$= eE - m\omega_o^2 x - \gamma v \quad (31)$$

$$m\dot{v} = eE - m\omega_o^2 x - \gamma v \quad (32)$$

Using displacement approach in equation (31), one can assume that.

$$x = x_o e^{-i\omega t}$$
$$v = \dot{x} = -i\omega x$$
$$\ddot{x} = i^2\omega^2 x = -\omega^2 x (33)$$
$$-m\omega^2 x = eE - k_o x - \gamma(-i\omega x)$$
$$-m\omega^2 x = eE - k_o x + i\gamma\omega x$$
$$m\omega^2 x + i\gamma\omega x = k_o x - eE$$
$$(m\omega^2 + i\gamma\omega - k_0)x = -eE$$
$$x = \frac{eE}{(k_0 - m\omega^2 - i\gamma\omega)} (34)$$

For simplification let ($k_0 = 0$) to get:

$$x = \frac{-eE}{(m\omega^2 + i\gamma\omega)} \times \frac{(m\omega^2 - i\gamma\omega)}{(m\omega^2 - i\gamma\omega)}$$
$$x = \frac{(-eE)(m\omega^2 - i\gamma\omega)}{(m^2\omega^4 + \gamma^2\omega^2)}$$
$$x = \frac{e(i\gamma\omega - m\omega^2)E}{(m^2\omega^4 + \gamma^2\omega^2)} (35)$$

Using the definition of electric moment one gets

$$p = enx = \frac{e^2 n(i\gamma\omega - m\omega^2)E}{(m^2\omega^4 + \gamma^2\omega^2)} = \chi_e E$$

Hence

$$\chi_e = \frac{e^2 n(i\gamma\omega - m\omega^2)}{(m^2\omega^4 + \gamma^2\omega^2)} (36)$$

Thus the effective mass is given by:

$$m^* = \frac{m}{1 + \chi_e}$$
$$m^* = \frac{m}{\left[1 + \frac{e^2 n(i\gamma\omega - m\omega^2)}{(m^2\omega^4 + \gamma^2\omega^2)}\right]}$$
$$m^* = \frac{m(m^2\omega^4 + \gamma^2\omega^2)}{m^2\omega^4 + \gamma^2\omega^2 + e^2 ni\gamma\omega - e^2 nm\omega^2} \qquad (37)$$

Using the velocity approach one gets

$$v = v_0 e^{-i\omega t}$$
$$v = \dot{x} = -i\omega x$$
$$\ddot{x} = \dot{v} = -i\omega v$$

Substituting this results into equation (31) one gets

$$-im\omega v = eE + \frac{k_0}{i\omega} v - \gamma v$$
$$eE = \left(\gamma + \frac{ik_0}{\omega} - im\omega\right) v$$
$$v = \frac{eE}{\gamma + \frac{ik_0}{\omega} - i\omega m}$$
$$v = \frac{e\omega}{\gamma\omega + ik_0 - im\omega^2} E (38)$$

Thus the current density can be given as

$$\square = nev = \frac{ne^2\omega}{\gamma\omega + ik_0 - im\omega^2} E (39)$$

Equating equation (39) to equation (17) one gets

$$-i\omega\chi_e = \frac{e^2 n\omega}{\gamma\omega + ik_0 - im\omega^2}$$
$$\chi_e = \frac{e^2 n}{k_0 - m\omega^2 - i\gamma\omega} (40)$$

Hence the effective mass is given by:

$$\mathrm{m}^* = \frac{\mathrm{m}}{1 + \chi_\mathrm{e}} (41)$$

$$m^* = \frac{m(k_0 - m\omega^2 - i\gamma\omega)}{k_0 - m\omega^2 - i\gamma\omega + e^2 n} \quad (42)$$

VI. Discussion

The concept of effective mass is very important in solid state physics as it reflects the effect of crystal field in the electrons. In this work the expression of effective mass relies heavily on the quantum expression of the free particle which is a function of the mass and the wave number [13]. The expression of the energy in which the electric and magnetic fields affect the particle through the wave number (k) only is equated with expression in which the fields affect the energy through the mass only as shown by equation (1). According to this version the effective mass is dependent on ordinary free mass and free wave number beside the medium wave number as shown by equation (2).

Relating the medium wave number to electric and magnetic permittivity equation (4), the effective mass is shown by equation (7) to depend on the electric and magnetic susceptibilities. One considers here the effect of electric susceptibility only. When only free thermal vibration is considered to affect equation (9), it shows that the electric susceptibility depends on the free as well as field vibration.

Equation (14) shows that when no electric dipoles exist the effective mass equals the ordinary mass. This confirms the proposed model, since when no field exist experiments and common sense shows that the effective mass equals the ordinary mass. It is very interesting also to note that the two approaches used lead to the same results. The first approach expresses the equation of motion in terms of the displacement, while the second depends on the velocity as shown in equations (8), (9) and (15).

When frictional force is considered in equation (20) the susceptibility is dependent on the frictional coefficient according to equation (24). It is very interesting to observe in equation (25) that the mass is complex quantity. This means, same of this mass can be transformed to thermal energy.

This confirms with pair production and nuclear fusion processes where same masses transform to energy. Again equation (25) shows that when no electric dipoles and friction exist the effective mass reduces to the ordinary mass. For free vibrating electron in a frictional medium, the equation of motion is given by equation (31) and equation (32). Equation (42) shows that the effective mass is dependent on friction as well as on stiffness. When no electric dipoles exist, the effective mass equals the ordinary mass as equation (42) indicates.

VII. Conclusion

The propose model shows that the effective mass is affected by the electric internal field through the electric susceptibility. When no electric field exists, the effective mass equals the ordinary mass. Thus, this model is none relativistic than the conventional one in which the effective mass vanishes, when no external field is applied.

References

[1]. J. Antula, Phys, Solid State Electron, Lett. A, 25 (1967) 309; 11 (1968) 106.
[2]. Zhou, Huan-Xiang. "Chemically driven motility of Brownian particles." Physical review letters 77.1 (1996): 194.
[3]. Georgi, Howard, and Sheldon L. Glashow. "Unity of all elementary-particle forces." Physical Review Letters 32.8 (1974): 438.
[4]. Adachi, Sadao. "GaAs, AlAs, and Al x Ga1− x as: Material parameters for use in research and device applications." Journal of Applied Physics 58.3 (1985): R1-R29.
[5]. Reiser, Martin. Theory and design of charged particle beams. John Wiley & Sons, 2008.
[6]. Luttinger, J. M. "Quantum theory of cyclotron resonance in semiconductors: General theory." Physical Review 102.4 (1956): 1030.
[7]. C.M. Vodenicharov, Phys. Status Solidi A, 42 (1977) 785.
[8]. Das, Saurya, Elias C. Vagenas, and Ahmed Farag Ali. "Discreteness of space from GUP II: relativistic wave equations." Physics Letters B 690.4 (2010): 407-412.
[9]. Häser, Marco, and Jan Almlöf. "Laplace transform techniques in Mo/ller–Plesset perturbation theory." The Journal of chemical physics 96.1 (1992): 489-494.
[10]. Caloz, Christophe, and Tatsuo Itoh. Electromagnetic metamaterials: transmission line theory and microwave applications. John Wiley & Sons, 2005.
[11]. O'Connell, R. F. "The equation of motion of an electron." Physics Letters A 313.5-6 (2003): 491-497.
[12]. Dumas, P. "The heavy-atom problem: a statistical analysis. II. Consequences of the a priori knowledge of the noise and heavy-atom powers and use of a correlation function for heavy-atom-site determination." Acta Crystallographica Section A: Foundations of Crystallography 50.4 (1994): 537-546.
[13]. Hu, Wayne, Rennan Barkana, and Andrei Gruzinov. "Fuzzy cold dark matter: the wave properties of ultralight particles." Physical Review Letters 85.6 (2000): 1158.

Elmardi.A. M. Ali." The Impact of the Electricsusceptibility on Electron Effective Mass." IOSR Journal of Applied Physics (IOSR-JAP) , vol. 11, no. 3, 2019, pp. 25-29.