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The Impact of Magnetic Susceptibility on Electron Effective Mass

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Abstract: *The efficiency of magnetic field (magnetic susceptibility) on electron effective mass was studies by derivation (mathematically) for the motion equations (velocity and acceleration). Considering that the electron presented in magnetic field, an equation for electron effective mass was found. When neglecting the field, the effective mass is returned back to the ordinary mass.*

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I. Introduction

Electron effective mass (which is often denoted m*) is the mass that it seems to have when responding to forces, or the mass that it seems to have when interacting with other identical particles in a thermal distribution. However, the value of effective mass depends on the purpose for which it is used, and can vary depending on a number of factors [1].

II. Magnetic Susceptibility

The magnetic susceptibility (χ_m) is one measure of magnetic properties of a material. The susceptibility indicates whether a material is attracted into or repelled out of a magnetic field, which in turn has implications for practical applications. Quantitative measures of the magnetic susceptibility also provide insights in to the structure of materials, bonding and energy levels. If the magnetic susceptibility is greater than zero, the substance is said to be paramagnetic and the magnetization of the substance is higher than that of empty space. If the magnetic susceptibility is less than zero, the substance is diamagnetic and it tends to exclude a magnetic field from its interior [2]. Mathematically it is the ratio of magnetization M(magnetic moment per unit volume) to the applied magnetizing field intensity H.

Magnetic susceptibility (χ_m) is a measure of the ability of a substance to be magnetized when exposed to an external magnetic field.

If (M) is the magnetization of the material and (H) is the applied a magnetic field, the functional relationship can be written as:

$$M = \chi H \qquad (1)$$

where χ is a dimensionless entity. For paramagnetic and diamagnetic materials χ is a constant, while for, ferromagnetic and antiferromagnetic materials χ depends on a number of variables including the strength of the applied field, the particle size and the magnetic history of the sample [3].

The cyclotron effective mass (m*) and the cyclone resonance mobility were obtained by the maximum and the half-width of the Lorentz fitting of the absorption profile as a function of magnetic field. The value of effective mass is approximately (0.19m0).

The effective mass is remarkably increased with field in high magnetic field [4]. The increase is about 4% in the range 15 – 40T, and partly explained by the band non-parabolicity that is calculated to be about 2%. The observed larger increase may be due to the effect of the strain in the quantum well layer as observed in (cyclotronresonance) on inversion layers on (p-type (Si)).

The observed filling factor dependence of the effective mass is most probably due to the carrier screening effect on the potential fluctuations. Although there are no donor impurities in the quantum well, there are residual ionized impurities in the well and the screening effect by carriers would be significant [5].

The scattering by remote impurities in the d-doped layers in Si, Ge and the interface fluctuations would be also affected by the carrier screening. The decrease of the density of states at the Fermi energy for the integer filling factor would reduce the effect of the screening of the Coulomb potential of impurity, and the cyclone resonance line width would thus be increased [6].

The cyclone resonance mass and the line-width are linked with each other by common origin. In fact, the effective mass and the cyclone resonance mobility showed similar oscillations in the present experiment [7].
At the highest energies of the valence band in many semiconductors (Ge, Si, GaAs) and the lowest energies of the conduction band in some semiconductors (GaAs ...), the band structure $E(k)$ can be locally approximated as [8].

$$E(k) = E_0 + \frac{\hbar^2 k^2}{2m^*} \qquad (2)$$

Where E(k) is the energy of an electron at wave vector k in that band, E_0 is a constant giving the edge of energy of that band, and m* is a constant (the effective mass).

It can be shown that the electrons placed in these bands behave as free electrons except with a different mass, as long as their energy stays within the range of validity of the approximation above. As a result, the electron mass in models such as the Drude model must be replaced with the effective mass. One remarkable property is that the effective mass can become negative, when the band curves downwards away from a maximum. As a result of the negative mass, the electrons respond to electric and magnetic forces by gaining velocity in the opposite direction compared to normal; even though these electrons have negative charge, they move in trajectories as if they had positive charge (and positive mass). This explains the existence of valence-band holes, the positive-charge, positive-mass quasiparticles that can be found in semiconductors [9]. In any case, if the band structure has the simple parabolic form described above, then the value of effective mass is unambiguous Unfortunately [10]. This parabolic form is not valid for describing most materials. In such complex materials there is no single definition of "effective mass" but instead multiple definitions, each suited to a particular purpose. The rest of the article describes these effective masses in detail [11].

III. Electron effective mass in the presence of magnetic field only

An electron can be accelerated by electric field and magnetic field of flux density (B). Its equation of motion is

$$m\chi = eE + Bev \qquad (3)$$

Consider now the displacement to be that of vibrating string [12]. Thus

$$x = x_0\, e^{-i\omega t}$$

$$v = \dot{x} = -i\omega x$$

Hence

$$\ddot{x} = -\omega^2 x$$

$$-m\omega^2 x = eE - iBe\omega x$$

$$(iBe\omega - m\omega^2)x = eE$$

$$x = \frac{eE}{(iBe\omega - m\omega^2)} \quad (4)$$

$$x = \frac{-e(m\omega^2 + Be\omega i)}{m^2\omega^2 + B^2e^2\omega^2} E \quad (5)$$

$$P = enx$$

$$P = \frac{-e^2(m\omega^2 + Be\omega i)n}{(m^2\omega^4 + B^2e^2\omega^2)} E = \chi_e E$$

$$\chi_e = \frac{-e^2(m\omega^2 + Be\omega i)n}{(m^2\omega^4 + B^2e^2\omega^2)} \qquad (6)$$

$$m^* = \frac{m}{1 + \chi_e}$$

$$m^* = \frac{m(m^2\omega^4 + B^2e^2\omega^2)}{(m^2\omega^4 + B^2e^2\omega^2) - e^2(m\omega^2 + Be\omega i)n} \qquad (7)$$

It is very interesting to note that when no electric dipole moments exist

$$n = 0$$

And

$$m^* = m$$

I.e. the effective mass is equal to the ordinary mass. One can also use the fact that:

$$v = v_0 e^{-i\omega t}$$

$$x = \int v dt = \frac{v_0 e^{-i\omega t}}{-i\omega}$$

$$x = \frac{iv}{\omega}$$

$$\ddot{x} = -i\omega v$$

$$mx = eE + Bev$$

$$-i\omega m v = eE + Bev$$

$$-(\mathrm{Be}+\mathrm{im}\omega)\mathrm{v}=\mathrm{eE}$$

$$\mathrm{m}^{*}=\frac{\mathrm{eE}}{-(\mathrm{Be}+\mathrm{im}\omega)} \qquad (8)$$

$$\square=\mathrm{nev}$$

$$\square=\frac{-\mathrm{ne}^{2}\mathrm{E}(\mathrm{Be}-\mathrm{im}\omega)}{(\mathrm{Be}+\mathrm{im}\omega)(\mathrm{Be}-\mathrm{im}\omega)}$$

$$\square=\frac{\mathrm{ne}^{2}(\mathrm{im}\omega-\mathrm{Be})}{(\mathrm{B}^{2}\mathrm{e}^{2}-\mathrm{m}^{2}\omega^{2})} \qquad (9)$$

$$\square=\frac{\partial \mathrm{P}}{\partial \mathrm{t}}=\chi_{\mathrm{e}}\mathrm{E}_{0}\frac{\partial \mathrm{e}^{-\mathrm{i}\omega\mathrm{t}}}{\partial \mathrm{t}}$$

$$\square=-\mathrm{i}\omega\chi_{\mathrm{e}}\mathrm{E}$$

$$\square=\frac{\mathrm{ne}^{2}(\mathrm{im}\omega-\mathrm{Be})}{(\mathrm{B}^{2}\mathrm{e}^{2}+\mathrm{m}^{2}\omega^{2})}\mathrm{E}=-\mathrm{i}\omega\chi_{\mathrm{e}}\mathrm{E}$$

$$\chi_{\mathrm{e}}=\frac{\mathrm{ne}^{2}(\mathrm{m}\omega^{2}+\mathrm{Bei})}{\omega(\mathrm{B}^{2}\mathrm{e}^{2}+\mathrm{m}^{2}\omega^{2})} \qquad (10)$$

$$\mathrm{m}^{*}=\frac{1}{1+\chi_{\mathrm{e}}}$$

$$\mathrm{m}^{*}=\frac{1}{1+\left[\frac{\mathrm{ne}^{2}(\mathrm{m}\omega^{2}+\mathrm{Bei})}{\omega(\mathrm{B}^{2}\mathrm{e}^{2}+\mathrm{m}^{2}\omega^{2})}\right]}$$

$$\mathrm{m}^{*}=\frac{\omega(\mathrm{B}^{2}\mathrm{e}^{2}+\mathrm{m}^{2}\omega^{2})}{\omega(\mathrm{B}^{2}\mathrm{e}^{2}+\mathrm{m}^{2}\omega^{2})+\mathrm{ne}^{2}(\mathrm{m}\omega^{2}+\mathrm{Bei})} \qquad (11)$$

IV. The Effective Mass in the presence of Magnetic Field and Resistive Friction Force

The force acts on the electron can be written as

$$\mathrm{m}\ddot{\mathrm{x}}=\mathrm{Bev}+\mathrm{eE}-\gamma\mathrm{v} \qquad (12)$$

$$-\mathrm{m}\omega^{2}\mathrm{x}=-\mathrm{i}\omega\mathrm{Bex}+\mathrm{i}\omega\gamma\mathrm{x}+\mathrm{eE}$$

$$\mathrm{x}=\mathrm{x}_{0}\,\mathrm{e}^{-\mathrm{i}\omega\mathrm{t}}$$

$$\mathrm{v}=\dot{\mathrm{x}}=-\mathrm{i}\omega\mathrm{x}$$

Hence:

$$\ddot{\mathrm{x}}=-\omega^{2}\mathrm{x}$$

$$-\mathrm{m}\omega^{2}\mathrm{x}=\mathrm{eE}-\mathrm{iBe}\omega\mathrm{x}+\mathrm{i}\gamma\omega\mathrm{x}$$

$$[(\mathrm{Be}\omega-\gamma\omega)\mathrm{i}-\mathrm{m}\omega^{2}]\mathrm{x}=\mathrm{eE}$$

$$\mathrm{x}=\frac{\mathrm{e}}{[(\mathrm{Be}-\gamma)\mathrm{i}-\mathrm{m}\omega]\omega}\mathrm{E} \qquad (13)$$

Also there is:

$$\mathrm{P}=\mathrm{enx}$$

$$\mathrm{P}=\mathrm{en}\left(\frac{\mathrm{e}}{[(\mathrm{Be}-\gamma)\mathrm{i}-\mathrm{m}\omega]\omega}\mathrm{E}\right)$$

$$\mathrm{P}=\frac{\mathrm{ne}^{2}}{[(\mathrm{Be}-\gamma)\mathrm{i}-\mathrm{m}\omega]\omega}\mathrm{E} \qquad (14)$$

And:

$$\mathrm{P}=\chi_{\mathrm{e}}\mathrm{E}$$

Then:

$$\chi_{\mathrm{e}}\mathrm{E}=\frac{\mathrm{ne}^{2}}{[(\mathrm{Be}-\gamma)\mathrm{i}-\mathrm{m}\omega]\omega}\mathrm{E}$$

Then electric susceptibility is:

$$\chi_{\mathrm{e}}=\frac{\mathrm{ne}^{2}}{[(\mathrm{Be}-\gamma)\mathrm{i}-\mathrm{m}\omega]\omega} \qquad (15)$$

Hence:

$$\mathrm{m}^{*}=\frac{1}{1+\chi_{\mathrm{e}}}$$

$$\mathrm{m}^{*}=\frac{1}{\left(1+\frac{\mathrm{ne}^{2}}{[(\mathrm{Be}-\gamma)\mathrm{i}-\mathrm{m}\omega]\omega}\right)}$$

$$m^* = \frac{[(Be-\gamma)i - m\omega]\omega}{[(Be-\gamma)i - m\omega]\omega + ne^2} \quad (16)$$

Also one can consider:

$$v = v_0 e^{-i\omega t}$$
$$v = \dot{x} = -i\omega x$$
$$\ddot{x} = \dot{v} = -i\omega v$$
$$m\ddot{x} = Bev + eE - \gamma v$$
$$eE = (\gamma - Be - i\omega)v$$
$$v = \frac{e}{\gamma - Be - im\omega}E \quad (17)$$

And the current density is:

$$\square = nev$$
$$\square = \frac{ne^2}{\gamma - Be - im\omega}E \quad (18)$$

When:

$$\square = \frac{\partial P}{\partial t} = \chi_e E_0 \frac{\partial e^{-i\omega t}}{\partial t}$$
$$\square = -i\omega\chi_e E$$
$$\square = \frac{ne^2}{\gamma - Be - im\omega}E = -i\omega\chi_e E$$

Then electric susceptibility is:

$$\chi_e = \frac{ne^2}{-i(\gamma - Be - im\omega)\omega}$$

Then electric susceptibility is:

$$\chi_e = \frac{ne^2}{(iBe - i\gamma - m\omega)\omega} \quad (19)$$

Hence:

$$m^* = \frac{1}{1+\chi_e}$$
$$m^* = \frac{1}{1 + \frac{ne^2}{(iBe - i\gamma - m\omega)\omega}}$$

So the electron effective mass is:

$$m^* = \frac{(iBe - i\gamma - m\omega)\omega}{(iBe - i\gamma - m\omega)\omega + ne^2} \quad (20)$$

V. Discussion

The presence of magnetic field affects the equation of motion according to equation (3). The effective mass is dependent on the magnetic field strength as well as on the number of electric dipoles as shown by equation (7). The effective mass equals to the ordinary one when no electric dipoles exist [13]. When magnetic and frictional force affect on the electron, the effective mass depend on magnetic flux density as well as friction coefficient (γ) [see equation (20)].

It is very interesting to note that in all cases, the effective mass reduces to the ordinary mass when the electric field is absent. Thus this model is more advanced than the effective vanishes and does not reduce to the ordinary rest mass.

VI. Conclusion

Hear it seems clearly, that the effective mass is affected by the magnetic field through the magnetic susceptibility. When no magnetic field exists, the effective mass equals the ordinary mass. Thus this model is none relativistic than the conventional one in which the effective mass vanishes, when no external field is applied.

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