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"The Dependence of Supersymmetric Structure on Temperature Using Enlarged Fock Space"

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Abstract: *It is shown that system has supersymmetry which is not broken at a given temperature by mean of thermofield dynamics. Using Fockspace, supersymmetric generators being constructed. Besides realizing breakdown of supersymmetry at a given temperature. What happen about broken of supersymmetry using Fockspace at absolute zero temperature? The comparative study of supersymmetry broken with "Van Hove" result in detail. The study also includes use of Lagrangian in thermofield dynamics [TFD]. How super charges explain the broken phenomenon of supersymmetry. As an analytical approach, the SUSY-QM approach has been utilized to study a number of problems in a quantum mechanics including the Morse oscillator [1] and the radial hydrogen atom equation [2].*

Key words: *Thermofield dynamics, Supersymmetric Algebra, Supercharges and Fockspace.*

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I. Introduction

The effect of temperature on supersymmetric structure has been studied [4-10]. Neverthless, the issue of whether supersymmetric structure is broken or not at given temperature and that cause some controversy. Earlier, it was observed that supersymmetric structure is unbroken at T=0K but broken at some positive temperature [13]. In this regard, when a change of an operator under SUSY transformation at given temperature is considered then Klein operator can be used. On introducing Klein operator, it was observed that the thermal average of this charge of an operator is zero for all temperature [5]. This issue has considered within the context of real time formalism or Thermo field dynamics (TFD) [7] whereas after some time it was concluded that SUSY is broken at given temperature using statistical average of SUSY Hamiltonian at absolute zero temperature as its vacuum expectation value is the"thermal vacuum"$|0\,(\beta)>$(where $\beta = \frac{1}{KT}$, here $K = Boltzmann\ constant$) and showing that it is non zero at given temperature [6,9]. In research paper [3] , examination of construction of SUSY algebra in TFD and understanding of SUSY at given temperature using Fock space.

It is the purpose of this work to understood and explain the supersymmetry structure using enlarging Fock space and "TILDE" operators and its dependence on temperature.

II. Thermo Field Dynamics

In TFD, the expectation value of an operator is equated to its statistical average, called " thermal vacuum". In doubled Fock space, thermal vacuum is temperature dependent. It leads to introduction of "TILDE" operators. Since double Fock space is a direct product of the two Fock spaces for non Tilde and Tilde creation and annihilation operators. This doubling nature is one of the most fundamental and universal feature of all the thermal quantum field formalism. For an ensemble of free Bosons with frequency ω, the Hamiltonian of the system is

$$H_B = \omega a^+ a \qquad \text{……..(1)}$$

Such that $$|a, a^+] = 1, \qquad \text{…….(2)}$$

Where a and a^+ are the annihilation and creation operators for Bosons.

Let us introduces the tilde fields by the Hamiltonian as

$$\bar{H}_B = \omega \bar{a}^+ \bar{a} \qquad \text{…….(3)}$$

with $$[\bar{a}, \bar{a}^+] = 1 \qquad \text{…....(4)}$$

such that $$[a, \bar{a}] = [a, \bar{a}^+] = 0, \qquad \text{…….(5)}$$

where $\bar{a}$ and $\bar{a}^+$ are the annihilation and creation operators for the tilde Bosonic fields. The thermal vacuum is given by

$$|0(\beta) >= [1 - \exp(-\beta\omega)]^{\frac{1}{2}} \exp(e^{-\frac{\beta\omega}{2}} a^+ \bar{a}^+)|0, \bar{0} >, \qquad \text{…….(6)}$$

where a doubling of Fock space is exhibited. The operators $a, a^+, \bar{a}, \bar{a}^+$ pertain to zero temperature. The corresponding operators at finite temperature, namely, $a(\beta), a^+(\beta), \bar{a}\,(\beta) and\ \bar{a}^+(\beta)$ are obtained from $a, a^+, \bar{a}, \bar{a}^+$ by Bogoliubov transformation. It is important to note that while

$$a|0(\beta)> \neq 0, \quad \text{………(7)}$$

we have

$$a(\beta)|0(\beta)> = 0 \quad \text{………(8)}$$

so that the Fock space at finite temperature is given by

$$|0(\beta)>, a^+(\beta)|0(\beta)>, \bar{a}^+(\beta)|0(\beta)>, \frac{1}{\sqrt{n!}}\frac{1}{\sqrt{m!}}\{a^+(\beta)\}^n\{\bar{a}^+(\beta)\}^m|0(\beta)^m|0(\beta)> \quad \text{……..(9)}$$

For an ensemble of Fermions of frequency ω (say), one has similar relations with the commutator replaced by anti-commutator, and the Fock space at finite temperature will be given by

$$|0(\beta)>, f^+(\beta)|0(\beta)>, \bar{f}^+(\beta)|0(\beta)>, f^+(\beta)\bar{f}^+(\beta)|0(\beta)> \quad \text{..(10)}$$

A physical interpretation of the doubling of the degrees of freedom, namely, $a, a^+$and $\bar{a}, \bar{a}^+$ for Bosons and / or f, f^+ and $\bar{f}, \bar{f}^+$ for Fermions are the following.

When the vacuum $|0(\beta)>$ is required to be independent of time, as it should be, we choose the total Hamiltonian for free Boson fields in TFD as

$$\bar{H}_B = \int d^3k\omega_k\{a_k^+a_k - \bar{a}_k^+\bar{a}_k\} \quad \text{…..(11)}$$

To an external stimulus, at $T \neq 0$, certain number of quantum particles are condensed in this system, which is in thermal equilibrium state with temperature T. The absorption of the external energy by the system results in [2] absorption by excitation of additional quanta and [5] excitation of quantum present in the vacuum, creating a hole. This is how one has doubling in TFD. It attribute the excitation of holes to that in the thermal reservoir.

In studying the properties of dynamical observables, it is expected to use the non-tilde operators. However, in studying the symmetry properties of the system, one needs both the tilde and non-tilde operators. This is emphasized 'in studying the spontaneous breakdown of $\bar{G}$ symmetry (that of the Bogoliubov transformation). In this Reference, the $\bar{G}$ symmetry is defined to be spontaneously broken when $\bar{G}|0(\beta)> \neq 0$ while $[\bar{G}, \bar{H}_o] = 0$. It is to be noted that the hat-Hamiltonian is used which has the tilde operators. For a system of free Fermions, the total Hamiltonian is

$$\bar{H}_F = \int d^3k\omega_k\{f_k^+f_k - \bar{f}_k^+\bar{f}_k\} \quad \text{…..(12)}$$

III. Supersymmetric Algebra:

With reference to [5], I expect from supersymmetry to have non-trival consequences not only at $T = 0$ but also at finite temperature, since all excited states must also some how reflect the supersymmetry property. In view of this we wish to examine the possibility of maintaining supersymmetry at finite temperature as well. By considering the enlarged Fock space (9) and (10) and using (11) and (12), We will in agreement of [5].

We demonstrate this by considering a system of free Bosons and free Fermions. At zero temperature, the Bosons are described by the creation and annihilation operators a^+ and a and the corresponding tilde operators satisfying the algebra

$$[a, a^+] = 1; \quad [a, a] = 0 \quad \text{….(13)}$$

$$[\bar{a}, \bar{a}^+] = 1, [\bar{a}, \bar{a}] = 0\,; [a, \bar{a}] = 0, [a, \bar{a}^+] = 0 \quad \text{….(14)}$$

and similarly the Fermions are described by $f, f^+, \bar{f}, \bar{f}^+$ satisfying the algebra

$$\{f, f^+\} = 1\,, \quad f^2 = (f^+)^2 = 0$$

$$\{\bar{f}, \bar{f}^+\} = 1\,, \quad \bar{f}^2 = (\bar{f}^+)^2 = 0$$

$$\{f, \bar{f}\} = (f, \bar{f}^+) = 0 \quad \text{……(15)}$$

We construct Fermionic (super) charge operators (generators of Supersymmetry) as

$$Q_+ = af^+; \quad Q_- = a^+f,$$

$$and \quad q_+ = \bar{a}\bar{f}^+; \ q_- = \bar{a}^+\bar{f}, \quad \text{…….(16)}$$

These operators are nilpotent, namely, $Q_+^2 = Q_-^2 = q_+^2 = q_-^2 = 0$ and convert Boson to Fermion and vice-versa, when acting on the representative state $|n_B, \bar{n}_B, n_F, \bar{n}_F>$. The elements of the superalgebra are

$$Q_\pm, q_\pm, (N_B + N_F), (\bar{N}_B + \bar{N}_F), \quad \text{……(17)}$$

where $N_B = a^+a, N_F = f^+f, \bar{N}_B = \bar{a}^+\bar{a}, \bar{N}_F = \bar{f}^+\bar{f}$. This algebra is closed

$$such\ that \quad \{Q_+, Q_-\} = N_B + N_F,$$

$$\{q_+, q_-\} = \bar{N}_B + \bar{N}_F,$$

$$\{Q_+, q_+\} = \{Q_-, q_-\} = \{Q_+, q_-\} = \{ Q_-, q_+\} = 0$$
$$[Q_\pm, (N_B + N_F)] = 0,$$
$$[Q_\pm, (\bar{N}_B + \bar{N}_F)] = 0,$$
$$[q_\pm, (N_B + N_F)] = 0,$$
$$[q_\pm, (\bar{N}_B + \bar{N}_F)] = 0, \qquad \text{.....(18)}$$

Satisfying the structure $\{O, O\} = E, [O, E] = E, [E, E] = E$ for even (E) and odd (O) operators. The total Hamiltonian for supersymmetric oscillator $\bar{H} = (a^+a - \bar{a}^+\bar{a} + f^+f - \bar{f}^+\bar{f}\ )$ is given by the anti-commutator

$$H = \{Q_+, Q_-\} - \{q_+, q_-\} \qquad \text{... ... ... (19)}$$

and $Q_\pm, q_\pm$ are Fermionic constants of motion, i.e.

$$[Q_\pm, \bar{H}] = [q_\pm, \bar{H}] = 0 \qquad \text{.......(20)}$$

The supersymmetric vaccum state at zero temperature is

$|0>= |0_B, \bar{0}_B, 0_F, \bar{0}_F>$ and since this vacuum is annihilated by $a, \bar{a}, f and\ \bar{f}\ and$ it follows that

$$< 0|\bar{H}|0 >= 0, \qquad \text{.....(21)}$$

and

$$Q_\pm|0 >= 0,$$
$$q_\pm|0 >= 0. \qquad \text{.....(22)}$$

Thus the supersymmetry constructed in (17) and (18) are not broken at absolute zero temperature.

IV. Supersymmetry at given temperature

At given temperature, we will exhibit a mathematical possibilities to examine whether supersymmetry is broken or not. In view of the structure of vacuum at finite temperature (9) and (10), let us choose the thermal vacuum for the supersymmetric case as

$$|0(\beta) >= |0_B(\beta), \bar{0}_B(\beta), 0_F(\beta), \bar{0}_F(\beta) > \qquad \text{.....(23)}$$

The zero-temperature operators $a, \bar{a}, f and \bar{f}$ are related to the 'temperature dependent' operators $a(\beta), \bar{a}(\beta), f(\beta)$ and $\bar{f}(\beta)$which annihilate the above 'thermal vacuum', by the (inverse) Bogoliubov transformation as

$$a = u(\beta)a(\beta) + v(\beta)a^+(\beta),$$
$$\bar{a} = u(\beta)\bar{a}(\beta) + v(\beta)\bar{a}^+(\beta),$$
$$f = U(\beta)f(\beta) + V(\beta)\bar{f}^+(\beta),$$
$$\bar{f} = U(\beta)\bar{f}(\beta) - V(\beta)f^+(\beta), \qquad \text{......(24)}$$

where

$$u(\beta) = \left(1 - e^{-\beta\omega}\right)^{-\frac{1}{2}}$$
$$v(\beta) = \left(e^{+\beta\omega} - 1\right)^{-\frac{1}{2}}$$
$$U(\beta) = \left(1 + e^{-\beta\omega}\right)^{-\frac{1}{2}},$$
$$V(\beta) = \left(1 + e^{+\beta\omega}\right)^{-\frac{1}{2}} \qquad \text{........(25)}$$

The construction of the supersymmetric charges $Q_\pm, q_\pm$ in (16) and $Q_\pm(\beta), q_\pm(\beta)$ in (22) in examining the supersymmetry breaking or not, using the total Hamiltonian $\bar{H}\ and\ \bar{H}(\beta)$ is on mathematical grounds, in the sense that in thermo field dynamics, the observables are analysed in terms of non-tilde operators while the above mathematical procedure includes tilde operators as well. It is still possible to realize unbroken supersymmetry at $T \neq 0$ without using the tilde operators by restricting the super algebra to

$$Q_\pm(\beta)\ and\ \{N_B(\beta) + N_F(\beta)\}.$$

This algebra is closed, namely,

$$\{Q_+(\beta), Q_-(\beta)\} = N_B(\beta) + N_F(\beta) \qquad \text{..........(26)}$$
$$[Q_\pm(\beta),\ \{N_B(\beta) + N_F(\beta)\}] = 0 \qquad \text{..........(27)}$$

The Hamiltoman of the system is identified with

$$H(\beta) = \{Q_+(\beta), Q_-(\beta)\} = a^+(\beta)a(\beta) + f^+(\beta)f(\beta) \quad \text{....(28)}$$

so that

$$[Q_\pm(\beta), H(\beta) = 0, \qquad \text{.....(29)}$$

giving the Fermionic constants of motion with respect to $H(\beta)$. The ground state is the thermal vacuum $|0(\beta) >$ as before. Then it follows:

$$< 0(\beta)|H(\beta)|0(\beta) >= 0,$$
$$Q_\pm(\beta), |0(\beta) >= 0 \qquad \text{......(30)}$$

showing that supersymmetry is not broken at finite temperature. The situation at zero temperature in this case is obtained by taking the limit $\beta \to \infty$. We have from equations (24) and (25),

$$u(\beta) \to 1; v(\beta) \to 0; a(\beta) \to a \; and \; f(\beta) \to f;$$

such that $H(\beta) \to H; |0(\beta) > \to |0 > as \; \beta \to \infty$ and then we recover the zero temperature case and this has supersymmetric structure unbroken.

V. Results and Discussion

From detail study of supersymmetric structure and their temperature dependence, it is clear that the construction of supersymmetric charges $Q_\pm$ and $q_\pm$ in equation (16) and $Q_\pm(\beta) \& q_\pm(\beta)$ in equation (22) which examine the supersymmetry breaking or not using the total Hamiltonian $\bar{H}$ and $\bar{H}(\beta)$ on mathematical grounds, in view of TFD. The observables are analysed in terms of nontilde operators while the above mathematical procedure includes tilde operators as well. It is still possible to realize unbroken supersymmetry at $T \neq 0$ without using the tilde operators by restricting the superalgebra to equation (26). Equation (31) showing that supersymmetry is not broken at finite temperature. The situation at zero temperature in this case is obtained by taking $\beta \to \infty$. From equations (24) and (25), we recover the zero temperature case and it has supersymmetric structure unbroken.

VI. Conclusion

In this research paper, I have examined temperature dependence of super symmetric structure in TFD on introducing the enlarged Fockspace. Here supersymmetric generators are being constructed in which Hamiltonion is governed by the anti commutator of supercharges. Besides, realizing spontaneous breakdown of supersymmetry. These have well defined zero temperature limit in which the supersymmetry is not broken, which is in agreement with [3,5]. Thus, we conclude that supercharge operators and enlarged Fock space capable to explain the broken and unbroken mechanism of Bosons and Fermions and their supersymmetry.

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