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The Simplest realistic model of interaction potentials in study of Generalized super symmetry

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Abstract: *A suitable model of interaction potentials can be used in detailed study of generalized supersymmetry using creation and annihilation operators in a quantum mechanics. The study of generalized supersymmetry are used in atomic physics, molecular physics, nuclear physics and particle physics. The formalism and the techniques of SUSY quantum mechanics is generalized to the cases where the super potential is generated/defined by higher excited eigen states. The SUSY formalism applies everywhere between the singularities. A systematic application of the formalism to the other potentials with known spectra would yield an infinitely rich class of "solvable" potentials in terms of their partner potentials.*

Keywords: *Creation and annihilation operators, generalized supersymmetry and superposition of supersymmetries.*

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I. Introduction

Now a days, study of interesting feature of quantum field theory using reduction of space time to a point takes place [4]. In its turn, the quantization recipe becomes non unique. Such a type of ambiguity was unexpected. A certain complexified version of quantum mechanics [7] obtained using one dimensional space time continuum. In such a slightly more realistic setting the anbiguity results from the indeterminate complex asymptotic boundary conditions [3, 12].

Within this fresh methodical framework, there is detailed study of traditional concept of quantum mechanics [1,7]. In an attempt to apply the PT symmetric formalism of the quantum mechanics of more particles [3,8,9], we were forced to return to the generalized supersymmetry. This provided a key motivation for our forthcoming discussion. The PT symmetric approach of the field theory opens new unresolved questions. The Witten's super symmetry in quantum mechanics [1, 5, 6] did also find its natural PT symmetrized new versions [12]. An explicit demonstration of their applicability to the generalized supersymmetry is still missing and will in fast be a core of our present work.

II. Creation And Annhilation Operators:

[A] Generalized supersymmetry:

By direct computations we reveal that in the vicinity of any semi integer parameter $\delta -$ the superscripted operators, given by

$$A^\delta = \partial_r + W^\delta \quad and \quad B^\delta = -\partial_r + W^\delta \qquad \text{.....(1)}$$

$$\text{where } \delta \neq 0, \pm 1, \ldots$$

act an our (normalized, spiked and PT-symmetrized) harmonic oscillator states in a transparent way guided by the preceding examples,

$$A^\delta L_{n+1}^\delta = C_1(n,\delta) L_n^{\delta+1}; where \quad C_1(n,\delta) = -2\sqrt{(n+1)} \qquad \text{.......(2)}$$

$$B^\delta L_n^{\delta+1} = C_2(n,\delta) L_{n+1}^\delta; where \quad C_2(n,\delta) = -2\sqrt{(n+1)} \qquad \text{.......(3)}$$

$$A^\delta L_n^{-\delta} = C_3(n,\delta) L_n^{-(\delta+1)}; where\ C_3(n,\delta) = 2\sqrt{(n-\delta)} \qquad \text{.......(4)}$$

and $$B^\delta L_n^{-(\delta+1)} = C_4(n,\delta) L_n^{-\delta}; where \quad C_4(n,\delta) = 2\sqrt{(n-\delta)} \qquad \text{.......(5)}$$

The former two rules were sufficient to define the above mentioned annihilation and creation at $\alpha = \frac{1}{2}$. The latter two lines have to be added in order to move us to $\alpha \neq \frac{1}{2}$. As operators which mediate the supersymmetric mapping they also import an explicit $\delta -$ dependence in C_3 and C_4. Its definitely most embarrassing consequence is the singularly at $\delta =$ integer which reflects the simultaneous (unavoided) crossing of the energy levels[3]. We shall again skip these points as exceptional.

[B] Superpositions of supersymmetries:

In the next investigative step, let us contemplate a superposition of the mappings sampled in Table 1.

Table 1: Superposition of mappings

$E_{\frac{L}{R}}$	$\mid n_L > \overset{A^{\frac{1}{2}}}{\rightarrow} \mid n_R >= \mid n_L > \underset{A^{-3/2}}{\longrightarrow} \mid n_R >$	$E_{\frac{\bar{L}}{\bar{R}}}$
…..	……. ……. ……. ……..	…..
…..	……. ……. ……. ……..	…..
8	$L_2^{\frac{1}{2}} \longrightarrow$ $L_1^{\frac{3}{2}} \longrightarrow$ $L_1^{\frac{1}{2}}$	10
6	$L_2^{-\frac{1}{2}} \longrightarrow$ $L_2^{-\frac{3}{2}} \longrightarrow$ $L_1^{-\frac{1}{2}}$	8
4	$L_1^{\frac{1}{2}} \longrightarrow$ $L_0^{\frac{3}{2}} \longrightarrow$ $L_0^{\frac{1}{2}}$	6
2	$L_1^{-\frac{1}{2}} \longrightarrow$ $L_1^{-\frac{3}{2}} \longrightarrow$ $L_0^{-\frac{1}{2}}$	4
0	$L_0^{\frac{1}{2}} \longrightarrow$ $0 \longrightarrow$ …	2
-2	$L_0^{-1/3} \longrightarrow$ $L_0^{-3/2} \longrightarrow$ 0	0

In the Hermitian limit $\in \rightarrow 0$, this table gives the explicit annihilation pattern for the s-wave spherical oscillator, after we drop a half of the PT symmetric solutions for the "external" physical reasons. The richer spectrum of this table with $\in \neq 0$ correspond to the supersymmetrically assigned pair of PT symmetrically regularized non Hermitian Hamiltonians $H_L = \left(H^{\frac{1}{2}} - 3\right)$ and $H_R = \left(H^{\frac{3}{2}} - 1\right)$ in combination with the similar partnership of the shifted second pair of

$H_{\bar{L}} = \left(H^{\frac{3}{2}} + 1\right)$ and $H_{\bar{R}} = \left(H^{\frac{1}{2}} + 3\right)$. This indicates the importance of the preserved shape invariance of the general PT regularized spiked harmonic oscillator.

For $\delta = -\frac{3}{2}$, PT supersymmetry between $H_L = (H^{\frac{3}{2}} + 1)$ and $H_R = (H^{\frac{1}{2}} + 3)$ would be followed by the $\delta = \frac{1}{2}$ correspondence between the doublet $H_{\bar{L}} = \left(H^{\frac{1}{2}} - 3\right)$ and $H_{\bar{R}} = \left(H^{\frac{3}{2}} - 1\right)$. The result is presented in Table 2.

PT regularized pattern of annihilation

Table 2: Singular Hamiltonian $H^{\frac{3}{2}} = p^2 + (x - i \in)^2 + \frac{2}{(x - i\in)^3}$ and the PT regularized pattern of annihilation

$E_{\frac{L}{R}}$	$\mid n_L > \underset{A^{-\frac{3}{2}}}{\longrightarrow} \mid n_R = \mid n_L > \overset{A^{\frac{1}{2}}}{\rightarrow} \mid n_R >$	$E_{\frac{\bar{L}}{\bar{R}}}$	
…..	……. ……. ……. ……..	…..	
…..	……. ……. ……. ……..	…..	
14	$L_2^{\frac{3}{2}} \longrightarrow$ $L_2^{\frac{1}{2}} \longrightarrow$ $L_1^{\frac{3}{2}}$	8	
12	$L_3^{-\frac{3}{2}} \longrightarrow$ $L_2^{-\frac{1}{2}} \longrightarrow$ $L_2^{-\frac{3}{2}}$	6	
10	$L_1^{\frac{3}{2}} \longrightarrow$ $L_1^{\frac{1}{2}} \longrightarrow$ $L_0^{\frac{3}{2}}$	4	
8	$L_2^{-\frac{3}{2}} \longrightarrow$ $L_1^{-\frac{1}{2}} \longrightarrow$ $L_1^{-\frac{3}{2}}$	2	
6	$L_0^{\frac{3}{2}} \longrightarrow$ $L_0^{\frac{1}{2}} \longrightarrow$ 0	0	
4	$L_1^{-3/2} \longrightarrow$ $L_0^{-1/2} \longrightarrow$ $L_0^{-\frac{3}{2}}$	-2	
2	… … … . . $\longrightarrow$ … . .	-4	
0	$\longrightarrow$ … … … ..	-6	
	$L_0^{-\frac{3}{2}} \longrightarrow$ $0 \longrightarrow$ …		

As a net result we obtained the appropriate generalization of the annihilation and creation pattern for the p-wave column. We see that a nice and transparent structure energies from all the similar PT supersymmetric assignments. One of their most interesting properties in the ambiguity of the annihilation, or, in a less confusing manner, by the comparison of Tables 2 and 3.

Table 3: The same singular Hamiltonian

$$H^{\frac{1}{2}} = p^2 + (x - 1 \in)^2 + \frac{2}{(x - i \in)^2}$$

and an alternative annihilation pattern

$E_{\frac{L}{R}}$	$\mid n_L > \underset{A^{\frac{3}{2}}}{\rightarrow} \mid n_R >= \mid n_L > \xrightarrow{A^{-\frac{5}{2}}} \mid n_R >$	$E_{\frac{\bar{L}}{\bar{R}}}$	
….. ….. 8 6 4 2 0 -2 -4 -6	……. ……. ……. …….. ……. ……. ……. …….. $L_2^{\frac{3}{2}} \longrightarrow L_1^{\frac{5}{2}} \longrightarrow L_1^{\frac{3}{2}}$ $L_3^{-\frac{3}{2}} \longrightarrow L_3^{-\frac{5}{2}} \longrightarrow L_2^{-\frac{3}{2}}$ $L_1^{\frac{3}{2}} \longrightarrow L_0^{\frac{5}{2}} \longrightarrow L_0^{\frac{3}{2}}$ $L_2^{-\frac{3}{2}} \longrightarrow L_2^{-\frac{5}{2}} \longrightarrow L_1^{-\frac{3}{2}}$ $L_0^{\frac{3}{2}} \longrightarrow 0 \longrightarrow \dots \dots$ $L_1^{-3/2} \longrightarrow L_1^{-5/2} \longrightarrow L_0^{-\frac{3}{2}}$ $\dots \dots \dots .. \longrightarrow \dots \dots \longrightarrow \dots \dots \dots ..$ $L_0^{-\frac{3}{2}} \longrightarrow L_0^{-\frac{5}{2}} \longrightarrow 0$	….. ….. 14 12 10 8 6 4 2 0	

The latter table displays the superposition of the $\delta = \frac{3}{2}, PT$ supersymmetry between $H_L = \left(H^{\frac{3}{2}} - 5\right)$ and $H_R = \left(H^{\frac{5}{2}} - 3\right)$, with the $\delta = -\frac{5}{2}$, the PT supersymmetry between $H_{\bar{L}} = (H^{\frac{5}{2}} + 3)$ and $H_{\bar{R}} = (H^{\frac{3}{2}} + 5)$.

We are near a climax of present study. Having spotted the difference between the regular and singular supersymmetries with $\alpha = \frac{1}{2}$ and $\alpha \neq \frac{1}{2}$, respectively, we can also easily move to the non integer values of 2α giving the non equidistance spectra. Unless we reach the point of degeneracy $\alpha =$integer, all our formulae remain applicable. The annihilation operators and their creation partners acquire the factorized, second order differential form, given by

$A^{-(\delta+1)}.A^{\delta} = A^{\delta-1}.A^{-\delta} = A(\alpha)$ …… (6)

and $B^{-\delta}.B^{\delta-1} = B^{\delta}.B^{-(\delta+1)} = A^{+}(\alpha)$ ……(7)

At any $\alpha \neq 0,1,2, \dots \dots$, they enable us to move along the spectrum of harmonic oscillator Hamiltonian H^{α}, given by

$A(\alpha)L_{n+1}^{\beta} = C_5(n,\beta)L_n^{\beta}$ ……(8)

and $A^{+}(\alpha)L_n^{\beta} = C_5(n,\beta)L_{n+1}^{\beta}$ ……(9)

where $C_5(n,\beta) = -4\sqrt{(n+1)(n+\beta)+1}\ and\ \beta = \pm\alpha$ …….(10)

We have achieved our final goal of a unified description of the spiked harmonic oscillators H^{α} within the PT symmetric framework. The general PT supersymmetric partnership has been shown mediated by the "shape invariance" operators A^{δ} and B^{δ}. At any non integer $\alpha > 0$, the role of the general creation and annihilation operators for a given, single Hamiltonian H^{α} has been shown played by their $\alpha -$ dependent and $\beta -$ preserving products $A^{+}(\alpha)$ and $A(\alpha)$, respectively.

III. Results and Discussion:

In this research paper, I shall try to implant a certain more satisfactory mathematical symmetry into the set of harmonic oscillator wave functions. What can we expect from moving to larger semintegers, $\propto = |\delta|$? Just a strengthening of the tendencies which were revealed in Table 1 and 2. There features can be simply extrapolated. Thus, a $\delta = \frac{3}{2}$ modification of table 1 will contain two more lines at its bottom. At the next positive, $\delta = \frac{5}{2}$ supersymmetry between $H_L = H^{5/2} - 7$ and $H_R = H^{\frac{7}{2}} - 5$ will introduce a ground state mapping i.e. $L_0^{-\frac{5}{2}} \xrightarrow{A^{\frac{5}{2}}} L_0^{-\frac{7}{2}}$. It appears at $E_{\frac{L}{R}} = -10$, witnessing just a continuing downward shift of the levels with the negative and decreasing superscripts.

Upto a constant shift, the PT supersymmetric partners coincide is given by the solution of the algebraic equation $|\delta| = |\delta + 1|$. This solution is unique $\left(for\ \delta = -\frac{1}{2}\right)$ and corresponds to the case where the poles in $A^{\delta}\ and\ B^{\delta}$ vanish. This is the only case tractable also without the use of the PT regularization.

A very exceptional role is played by the integers limits of $\propto$. In contrast, no special attention must be paid to the limit of vanishing spike $\left(\propto\rightarrow\frac{1}{2}\right)$. In general, with $\propto\neq\frac{1}{2}$, our PT regularization ($\in\neq 0$) can also be removed, if needed, via the limiting transition ($\in\rightarrow 0$) accompanied by the necessary halving of the axis of co-ordinates. This means that we have to replace $r(x) = (x - i\in)$ by the radial and real $r \in (0, \infty)$ and cross out all the states with $\beta < 0$. They are simply proclaimed "in acceptable" in the light their conventional interpretation.

IV. Conclusion

The formalism enables us to construct the creation and annihilation operators. We show, how the real spectrum complies with the current SUSY-type is expectrality in an unusual way.

In this research paper we conclude that the normalizable ground state exists for both H_L and H_R such that

$$E_{L0}^{-} = E_{R0}^{-} = -2$$

and the first excited state remains unmatched by any R-subscripted partner at

$E_L^{+} = 0$ as $excepted$. The general supersymmetric partnership has been shown mediated by the "shape in variance" operators A^{δ} and B^{δ} . At any non integer $\propto = 0$, the role of general creation and annihilation operators for a given, single Hamiltonian H^{α} has been shown played by their $\propto -$ dependent and $\beta -$ preserving products $A^{+}(\propto)$ and $A(\propto)$ respectively.

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