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Controlled Not (Cnot) Gate For Two Qutrit Systems

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Abstract: *Quantum computation applications utilize basically the structures having two definite states named as QuBit (Quantum Bit). In spin based quantum computation applications like NMR and EPR techniques, besides the Spin-*1/2 *systems (electron and proton spins), there are many systems having spins greater than* 1/2*. For instance, in some structures, coupled two electrons can form triplet states with total spin of* 1*, and* ^{14}N *nucleus has, one of the widely encountered nuclei, also spin of* 1*. In quantum computation, such systems are called Qutrit systems and have the potential of use in spin based quantum computation applications. Besides the well-known CNOT gates in QuBit system, we suggest some CNOT gates for Qutrit systems composed of two Spin-*1 *systems, forming control and target Qutrits.*

Keywords: *Qutrit, spin, quantum computing, CNOT*

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I. Introduction

In quantum computation systems, the basic gates like Hadamard, Pauli X, Y and Z, CNOT, phase shift, SWAP, Toffoli and Fredkin have been constructed for two-state systems. Quantum logical gate such as the SWAP gate, controlled phase gate [1–2], and CNOT gate [3-5], one of the essential building block quantum computers [5-6]. In spin based quantum computation applications, spin- 1/2 particles like proton and electron are the pioneering structures for two-state systems whose base states are represented as $|0\rangle, |1\rangle$. There are other commonly encountered systems with spin-1 forming three states known as qutrit systems whose basis states are shown as $|0\rangle, |1\rangle, |2\rangle$. For example, coupled two electrons form triplet states (with total spin of 1) and the nuclear spin of ^{14}N is also 1 and both systems form qutrit systems. Nuclear magnetic resonance (NMR) and electron paramagnetic resonance (EPR) spectroscopies, being spin based quantum computation application candidate instruments, already use qubit systems and/or are capable of using spins greater than 1/2. If basic quantum gates for spin-1 systems are constructed, they probably will become another alternative candidate for spin based quantum computation applications. One of the basic gates for spin-1 systems is CNOT gate, which requires two spin-1 particles where one of them controls qutrit and the other one controls target qutrit, similar to the CNOT gate for spin-1/2 systems. The control qutrit does not change after CNOT operation and only target qutrit is altered. In this work, besides the known CNOT gates of spin-1 systems, some other CNOT candidates will be suggested.

II. Material And Methods

The processors in which the status of any unit depends on the status of the other cluster are called controlled processors. A quantum circuit is a combination of quantum gates that are located in various rows and connected by quantum cables. CNOT gate is similar to qubit systems with a base including two qutrits as $|m\rangle\,|n\rangle$, where *m* is control qutrit and *n* is target qutrit [7]. Both *m* and *n* are represented by the numbers 0, 1 and 2, and therefore a total of nine unique states are obtained. CNOT gates must satisfy the condition [8].

$$|m\rangle\,|n\rangle = |m\rangle\,|n \oplus m\rangle \quad (1)$$

where the symbol $\oplus$ represent $(n - m)$ modulo d, and where *d* is the dimension of the Hilbert space spanned by qutrit system and here *d* = 3. Just like in qubit systems, the symbol $\oplus$ represents classical XOR logic gate. In the following discussion, the abstract expression $|m\rangle\,|n\rangle = |mn\rangle$ of CNOT gates for qutrit systems will be used.

The CNOT gate is a reversible gate and can be reversed by applying the same gate. On the other hand, the operator is unitary and unitary operators live the size of the operant they change intact [9].

Quantum logic gates for bases greater than two-dimensional qubit systems, namely qutrits and higher dimensional bases are relatively new subject for spin based quantum computation theory and applications because present discussions and formulations on the subject have focused on qubit systems [10]. Gates such as Hadamard, CNOT, SWAP, do not have classical equivalents. For example, the Hadamard gate makes it a superposition with other states of the system [11, 12-13]. In this work, five CNOT gate candidates were suggested whose elements were different combinations of two qutrit bases.

If the status of any well-known qubit operator is dependent on the status of another qubit, these operator are called controlled operators. The CNOT gate is the most commonly known of these types of operator s. When the status of the control qubit is reversed by the CNOT operator control qubit is $|1\rangle$, the status of the control qubit is $|0\rangle$, leaving the state of the target qubid unchanged.

When a quantum system $|ab\rangle$ of two qubits is taken into consideration, the operation is writes as $\mathrm{CNOT}_a|ab\rangle = |a\ a \oplus b\rangle$, where a is control qubit and b is target qubit. In the equation $\oplus$ operation is the process of summing by mode 2, namely XOR gate.

The matrix operator of the CNOT gate for two cubit systems is given as [14],

$$\mathrm{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (2)$$

Here the input state $|ab\rangle$ is CNOT gate with expression $|a\ a \oplus b\rangle$ where control qubit a does not change its state. One CNOT gate candidate implementation was proposed by Cory *et al*, [15, 16].

Spin based quantum computation applications are suitable for qutrit bases as well as qubit bases since different nuclear spins of atoms span higher dimensional Hilbert base spaces, and therefore the theoretical backgrounds will be necessary for these bases as well. Although some quantum gates have been proposed for spin-1 and higher, some more discussions and work seem to be necessary.

III. Result and Discussion

Two qutrits or two spin-1 systems form nine different states. In the expression $|\boldsymbol{m}\rangle\,|\boldsymbol{n}\rangle = |\boldsymbol{mn}\rangle$, where *m*, *n* = 0, 1, 2, the nine different states are constructed as $|\mathbf{00}\rangle$, $|\mathbf{01}\rangle$, $|\mathbf{02}\rangle$, $|\mathbf{10}\rangle$, $|\mathbf{11}\rangle$, $|\mathbf{12}\rangle$, $|\mathbf{20}\rangle$, $|\mathbf{21}\rangle$, $|\mathbf{22}\rangle$. The left qutrit in Equation 1 is control qutrit and does not change, and the target qutrit is altered after CNOT operation obeys the condition given in Equation 1.

Table 1: The suggested CNOT gates. All states are represented by $|mn\rangle$ where *m* is control and *n* is target qutrit.
The rows indexed 3 to 8 satisfy the modular arithmetic *m–n* mod 3 condition as discussed in the text.

Q_0	CNOT_A	CNOT_B	CNOT_C	CNOT_D
$\|00\rangle - 0$	$\|00\rangle - 0$	$\|00\rangle - 0$	$\|00\rangle - 0$	$\|00\rangle - 0$
$\|01\rangle - 1$	$\|01\rangle - 1$	$\|01\rangle - 1$	$\|01\rangle - 1$	$\|01\rangle - 1$
$\|02\rangle - 2$	$\|02\rangle - 2$	$\|02\rangle - 2$	$\|02\rangle - 2$	$\|02\rangle - 2$
$\|10\rangle - 3$	$\|10\rangle - 7$	$\|11\rangle - 8$	$\|12\rangle - 4$	$\|11\rangle - 5$
$\|11\rangle - 4$	$\|11\rangle - 8$	$\|10\rangle - 6$	$\|11\rangle - 5$	$\|12\rangle - 3$
$\|12\rangle - 5$	$\|12\rangle - 6$	$\|12\rangle - 7$	$\|10\rangle - 3$	$\|10\rangle - 4$
$\|20\rangle - 6$	$\|20\rangle - 4$	$\|21\rangle - 5$	$\|22\rangle - 7$	$\|21\rangle - 8$
$\|21\rangle - 7$	$\|22\rangle - 5$	$\|20\rangle - 3$	$\|21\rangle - 8$	$\|22\rangle - 6$
$\|22\rangle - 8$	$\|21\rangle - 3$	$\|22\rangle - 4$	$\|20\rangle - 6$	$\|20\rangle - 7$

Table 1 demonstrates four CNOT gate operations for two qutrit systems. Theoretically modular arithmetic is used to set up CNOT gates, (in qubit system classical XOR logic stands for modular arithmetic). Similarly, the suggested gate operations given in Table 1 and corresponding explicit operator matrices given in Equation 3 were obtained. In Table 1, the leftmost column is input states and other columns with titles from CNOT_A to CNOT_D are the results of corresponding CNOT gates. As will be seen in Table 1, except first three rows, the operations all satisfy modular arithmetic.

$$\mathrm{CNOT}_A = \begin{bmatrix} 1&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&1&0&0&0&0&0 \end{bmatrix} \quad \mathrm{CNOT}_B = \begin{bmatrix} 1&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&1&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0 \end{bmatrix}$$

$$\mathrm{CNOT}_C = \begin{bmatrix} 1&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0&0 \end{bmatrix} \quad \mathrm{CNOT}_D = \begin{bmatrix} 1&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&1&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&1&0 \end{bmatrix} \quad (3)$$

The determinants of all gates are common and unity. Some other properties and the interrelations of the suggested CNOT gates were given in Equation 3. It is easily seen that the squares of the gates and the inverses of the gates are interrelated. These properties indicate that the four CNOT gates can be generated from each other. A problem emerges at this point about the uniqueness of the four gates under consideration and uniqueness property needs to be considered separately. The results obtained from the transactions with other matrices are given in Table 2.

Table 2: Results obtained from the transactions with other matrices of CNOT gate for two qutrit systems. Here $\mathbb{1}_9$ represents unit matrix of dimension 9.

$(\mathrm{CNOT_A})^2 = \mathrm{CNOT_D}$	$(\mathrm{CNOT_A})^{-1} = \mathrm{CNOT_B}$	$(\mathrm{CNOT_B})^3 = \mathbb{1}_9$
$(\mathrm{CNOT_B})^2 = \mathrm{CNOT_C}$	$(\mathrm{CNOT_B})^{-1} = \mathrm{CNOT_A}$	$(\mathrm{CNOT_C})^3 = \mathbb{1}_9$
$(\mathrm{CNOT_C})^2 = \mathrm{CNOT_D}$	$(\mathrm{CNOT_C})^{-1} = \mathrm{CNOT_D}$	$(\mathrm{CNOT_D})^3 = \mathbb{1}_9$
$(\mathrm{CNOT_D})^2 = \mathrm{CNOT_C}$	$(\mathrm{CNOT_D})^{-1} = \mathrm{CNOT_C}$	

IV. Conclusion

In contrast to the qubit systems, the theoretical basis for qutrit systems and hence the basic gates have drawn attention during the recent years. Especially spin based quantum computation has the potential of using spins greater than 1/2; e.g. NMR and specifically EPR spectroscopic techniques cover different spins. In this theoretical study, four controlled-NOT (CNOT) gates for two qutrit systems were suggested which were formed through different permutations of similar operators. The operators obey the modular arithmetic rule $|m\rangle\,|n\rangle = |m\rangle\,|n \oplus m\rangle$, where m is control qutrit and n is target qutrit. Since the base system is qutrit, that is spin-1, the operators had to be nine dimensional. Some properties and discrepancies were discussed.

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