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New Exact solution of the non-relativistic Schroedinger equation for Central singular potential $V(r) = \alpha r^2 + \frac{\beta}{r^4} + \frac{\lambda}{r^6}$

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Abstract
The new exact bound state solutions of the Schroedinger equation for the central singular potential given by $V(r) = \alpha r^2 + \frac{\beta}{r^4} + \frac{\lambda}{r^6}$
are obtained by using a suitable ansatz. Here α, β *and* λ *are parameters of the given potential and are also obtained. For each solution, there is a separate relation, interrelating the parameters of the potential and orbital angular momentum quantum no.* l*. The eigenfunctions obtained here are in closed form and normalizable. Such solution is relevant in connection with quark model of hadrons and some other branches of physics like quarkonium physics.*
Key words*: Schroedinger wave equation, Singular potential, Exact solution.*

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I. Introduction

One of the important issues of quantum mechanics is to solve the non relativistic Schroedinger equation for the potential of physical interest. Unfortunately, however, only for a few potentials, the Schroedinger equation is found to be exactly solvable. But in recent years, a lot of attention has been paid towards obtaining exact solution of the Schroedinger equation for the potential of physical importance [6-8,11]. Today, it is although possible to solve the Schroedinger equation for any potential to a desired degree of accuracy, with the help of modern computers. An exact solution, however, have an aesthetic appeal. Further, the exact solutions can serve as bench mark to test the accuracy of various nonperturbative methods.

In present paper, we obtain a set of exact bound state solutions which consists eigen values and corresponding eigen functions of the Schroedinger equation for the Central singular potential given by

$$V(r) = \alpha r^2 + \frac{\beta}{r^4} + \frac{\lambda}{r^6} \quad \text{.......(1)}$$

The inverse power potential is relevant in atomic, molecular and nuclear physics. The photo decay of excited states and the radiationless decay can also be described by an inverse power potential [9-10].

II. Schroedinger equation and Exact solution

Consider the reduced Schroedinger equation (take h=1 = 2 m)

$$\Phi'' + \left[\mathrm{E} - \mathrm{V(r)} - \frac{l(l+1)}{\mathrm{r}^2} \right] \Phi = 0 \quad \dots \dots \dots (2)$$

The central singular potential is\

$$V(r) = \alpha r^2 + \frac{\beta}{r^4} + \frac{\lambda}{r^6}$$

If R(r) be the radial part of solution of Schroedinger wave equation (2)

then $R(r) = \frac{1}{r}\Phi(r)$ ... ... (3)

Consider the ansatz

$$\Phi(r) = exp\left\{\frac{1}{2}ar^2 + \frac{1}{2}br^{-2} + \frac{1}{2}cr^{-4}\right\}\sum_{m=0}^{\infty} a_m r^{2m+\nu} \quad \dots .(4)$$

where a,b, c and ν are constants to be chosen suitably.

Using equation (4) in equation (2), we see that the expansion coefficients a_m must satisfy the three term recurrence relation,

$$A_m a_m + B_{m+1}a_{m+1} + C_{m+2}a_{m+2} = 0 \quad \dots (5)$$

where $A_m = E + (4m + 2\nu + 1)a$ $\quad ..(6a)$

$$B_m = \{ (2m + \nu)(2m + \nu - 1) - l(l+1) + 2ab\} \quad \ldots.(6b)$$

and $C_m = b(4m + 2\nu - 3) - \beta$ …..(6c)

and we have set

$$a^2 = \alpha \quad \ldots.(7)$$

and $b^2 = \lambda$ ….(8)

It is clear that R(r) is finite for if

$$a = -\sqrt{\alpha} \quad \ldots\ldots(9)$$

and $b = \sqrt{\lambda}$ ……(10)

Now as a_0 is the first nonvanishing co efficient in equation (4), then from equation (5), we have

$$C_o = 0 \quad \ldots\ldots.(11)$$

$$\Rightarrow \nu = \frac{1}{2}\left[\frac{\beta}{\sqrt{\lambda}} + 3\right] \quad \ldots\ldots.(12)$$

Now, if $a_p \neq 0$, but $a_{p+1} = a_{p+2} = \cdots \ldots. = 0$, then equation (5) gives

$$A_p = 0 \quad \ldots\ldots..(13)$$

$$\Rightarrow E_p = \sqrt{\alpha}\left[4p + \frac{\beta}{\sqrt{\lambda}} + 4\right] \quad \ldots\ldots..(14)$$

Also, for a non trivial solution, A_m, B_m & C_m must satisfy the condition

$$\begin{vmatrix} B_o \; C_1 \; 0 \; 0 \; 0 \ldots..0 \\ A_o \; B_1 \; C_2 \; 0 \; 0 \ldots..0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \\ 0 \; 0 \ldots\ldots\ldots.. A_{p-1} B_p \end{vmatrix} = 0 \quad \ldots(15)$$

The various solutions can be generated by setting p=0, 1, 2, 3 …….etc.

There are two cases:

Case-I : **For p=0**, then by equation (14), the energy eigen value is given by

$$E_0 = \sqrt{\alpha}\left[\frac{\beta}{\sqrt{\lambda}} + 4\right] \quad \ldots..(16)$$

and the corresponding eigen function is given by

$$R(r) = a_0\left[\exp\left\{-\frac{1}{2}\sqrt{\alpha}\, r^2 + \frac{1}{2}\sqrt{\lambda}\, r^{-2}\right\}\right] r^{\frac{1}{2}\left(\frac{\beta}{\sqrt{\lambda}}+1\right)} \quad \ldots..(17)$$

On simplifying equations (9), (10) and (12) with equation (15), we have

$$\frac{1}{4}\left[\frac{\beta}{\sqrt{\lambda}} + 3\right]\left[\frac{\beta}{\sqrt{\lambda}} + 1\right] - l(l+1) - 2\sqrt{\alpha\lambda} = 0 \quad \ldots.(18)$$

This is an interrelation between parameters of potential and orbital angular momentum quantum no. l.

Case-II: **For p=1** : then by equation (14), the energy eigen value is given by

$$E_1 = \sqrt{\alpha}\left(\frac{\beta}{\sqrt{\lambda}} + 8\right) \quad \ldots..(19)$$

The condition (4) in conjunction with the equations (6a), (9), (10) and (12) , we have $R(r) = a_0\left[\exp\left\{-\frac{1}{2}\sqrt{\alpha}\, r^2 + \frac{1}{2}\sqrt{\lambda}\, r^{-2}\right\}\right] r^{\frac{1}{2}\left(\frac{\beta}{\sqrt{\lambda}}+1\right)}(a_o + a_1 r^2)$ …..(20)

On simplifying equations (9), (10), (12) and (15), we have

$$4aa_0 - \left\{\left(\frac{\beta}{2\sqrt{\lambda}} + \frac{7}{2}\right)\left(\frac{\beta}{2\sqrt{\lambda}} + \frac{5}{2}\right) - l(l+1) - 2\sqrt{\alpha\lambda}\right\} a_1 = 0 \quad \ldots(21)$$

This gives an interrelation between the parameters of potential and orbital angular momentum quantum no. l.

Continuing this way, we can generate a set of bound exact solutions for higher values of p.

III. Results and Discussion

The explicit expression of energy eigenvalue and eigenfunction are obtained for each solution. These solutions are valid when for, in general, each solution has an interrelation between the parameters of the potential and the orbital angular momentum quantum no. l. These solutions, besides having an aesthetic appeal, can be used as benchmark to test the accuracy and reliability of nonperturbative methods, which some times yield wrong result of solving the Schroedinger equations. The central singular potential considered in this problem may be relevant in study of quark model of hadrons.

IV. Conclusion

The exact bound state solutions of non relativistic Schroedinger equation for central singular potential $V(r) = \left(\alpha r^2 + \frac{\boldsymbol{\beta}}{\boldsymbol{r^4}} + \frac{\boldsymbol{\lambda}}{\boldsymbol{r^6}}\right)$ by using a suitable ansatz have been obtained. The exact solution consists eigenvalues and corresponding eigenfunctions with an interrelation between parameters α, β *and* λ of potential and orbital angular momentum quantum no. l. The eigenfunction which are obtained in closed form are also normalizable. Such exact bound state solutions plays most important role in atomic physics, molecular physics and nuclear physics, especially in quarkonium physics.

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