

## A Mathematical Analysis and Prediction of Carbon Dioxide Emission in Assam, West Bengal and Orissa, India

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**Abstract:** Global warming is one of the most important environmental curses ever to face with human society during last few decades. Undoubtedly, it is fatal for the existence of life and most importantly green house gases are mostly responsible for such global warming. Among the green house gases evolved from different sources, carbon dioxide is the most damaging gas for pollution and global warming. Increased emissions of carbon dioxide from different sources to the environment are much alarming due to rapid industrialisation, agriculture and related waste products. In this paper a mathematical model has been developed for carbon dioxide emission connected to three different states of the eastern and north eastern part of India, namely, West Bengal, Assam and Orissa upon application of non-linear regression method based on historical data of about 21 years. Afterwards, the paper predicts the long term evolutionary trend of the gas emission upon utilization of Instantaneous Rate of Change in the subjected states along with a comparative study of carbon dioxide emission in the states and it also presents prediction of carbon dioxide emission for few years.

**Keywords:** Global warming, Regression analysis, Least square method, Instantaneous Rate of Change (IROC), Green House Gases (GHG).

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### I. Introduction

During the last century, there has been a serious change in climate of the world because of global warming that has become a nightmare to scientists, researchers and academicians throughout the world as this change leads to a fatal situation for the existence of life on this planet. Increased emission of different green house gases are responsible for this alarming situation. Amongst them, an uncontrolled emission of carbon dioxide (CO<sub>2</sub>) from different sources is supposed to be the most important one (Houghton and Ding, 2001; Battle et al., 2010).

CO<sub>2</sub> remains in the atmosphere longer than the other major heat-trapping gases emitted as a result of human activities. After a pulse of CO<sub>2</sub> is emitted into the atmosphere, 40% will remain in the atmosphere for 100 years and 20% will retain for 1000 years, while the final 10% will take 10,000 years to turn over (Mukhopadhyay and Forsell, 2002). This literally means that the heat-trapping emissions we release today from our cars and power plants are setting the climate our next generation will inherit (Nag and Parikh, 2005). Increased rate of emission of this gas contributes a major share of global warming. The concentration of CO<sub>2</sub> in the earth was very low initially and it was about 280±10 parts per million by volume (ppmv) in 1750 (Basak and Nandi, 2014). Instead, due to rapid industrialization, population explosion, globalization, deforestation enormously increase its amount in the global atmosphere which ultimately leads to global warming (Marland et al., 1999). The global warming has serious implications for all aspects of human life, including infectious diseases. It is critically linked to the process of development and economic growth of country (Murthy et al., 1997). In India, rate of emission of CO<sub>2</sub> is also much alarming. India is the fourth largest emitter of CO<sub>2</sub> in the world (Source: Oak Ridge National Laboratory (ORNL), USA). Different states in India are also responsible for this unexpected emission of carbon dioxide. Authors Basak and Nandi, 2014, also made a state wise analytical study regarding total CO<sub>2</sub> emission. The study of Asadoorian et al., 2006, Jin et al., 2010, and Tokos and Xu, 2009 explored third degree polynomial model for the emission of Green House Gases (GHG). Among several identified states in India, West Bengal is one of the major CO<sub>2</sub> emitter.

In this paper, we have formulated mathematical model depending on CO<sub>2</sub> emission in some eastern states in India using least square method. The states considered here are West Bengal, Orissa and Assam using the data set of CO<sub>2</sub> emission for twenty one year's from 1980 to 2000. Emission data are extracted from different literatures of Ghoshal and Bhattacharyya, 2008 and Marland et al., 1999.

**II. Data And Model Formulation**

The data of CO<sub>2</sub> emission for the 18 years (1980-1997) is utilized for the purpose of modelling. We formulated a third degree polynomial model for the analysis of CO<sub>2</sub> emission of West Bengal, Assam and Orissa. For generating the model of CO<sub>2</sub> emission, we followed the work of Tokos and Xu (2009) and Basak and Nandi (2014). Authors suggested a third degree polynomial model for emission of CO<sub>2</sub> viz.

$$(1) \quad y = a + bx + cx^2 + dx^3 \quad \dots$$

where Z is the emission of carbon dioxide in metric ton and x is represents time in years.

With the help of the given data  $\{(x_1, y_1), (x_2, y_2), \dots \dots (x_n, y_n)\}$ , we may define the error associated by

$$(2) \quad F(a, b, c, d) = \sum_{i=1}^n (y_i - a - bx_i - cx_i^2 - dx_i^3)^2 \quad \dots$$

is a function of four variables a, b, c and d.

For minimization of error and to estimate corresponding a, b, c and d, we use multivariate calculus namely,

$$\frac{\partial F}{\partial a} = 0, \quad \frac{\partial F}{\partial b} = 0, \quad \frac{\partial F}{\partial c} = 0, \quad \frac{\partial F}{\partial d} = 0$$

That leads to the equations

$$\begin{aligned} -2 \sum (y_i - a - bx_i - cx_i^2 - dx_i^3) &= 0 \\ -2 \sum x_i (y_i - a - bx_i - cx_i^2 - dx_i^3) &= 0 \\ -2 \sum x_i^2 (y_i - a - bx_i - cx_i^2 - dx_i^3) &= 0 \\ -2 \sum x_i^3 (y_i - a - bx_i - cx_i^2 - dx_i^3) &= 0 \end{aligned}$$

Corresponding normal equations are,

$$\begin{aligned} \sum y_i &= na + b \sum x_i + c \sum x_i^2 + d \sum x_i^3 \\ \sum y_i x_i &= a \sum x_i + b \sum x_i^2 + c \sum x_i^3 + d \sum x_i^4 \\ \sum x_i^2 y_i &= a \sum x_i^2 + b \sum x_i^3 + c \sum x_i^4 + d \sum x_i^5 \\ \sum x_i^3 y_i &= a \sum x_i^3 + b \sum x_i^4 + c \sum x_i^5 + d \sum x_i^6 \end{aligned} \quad \dots$$

(3)

For given set of points  $(x_i, y_i); (i=1, 2, \dots, n)$ , the equations (3) are solved for a, b, c, d to evaluate estimated  $a^*, b^*, c^*$  and  $d^*$ . It has been found that in all the cases, the value of the 2nd order derivatives evolves to be positive at the points a, b, c, d. These establish minimization of F.

The third degree fitted polynomial of CO<sub>2</sub> emission is now estimated as

$$y = a^* + b^*x + c^*x^2 + d^*x^3 \quad \dots (4)$$

**III. Instantaneous Rate Of Change (IROC) Of Emission**

For the computation of rate of change of the gas, the derivative of equation (4) is presented as

$$(5) \quad \frac{dy}{dx} = b^* + c^*x + d^*x^2 \quad \dots$$

The equation (5) at a particular time is utilized for prediction of CO<sub>2</sub>.

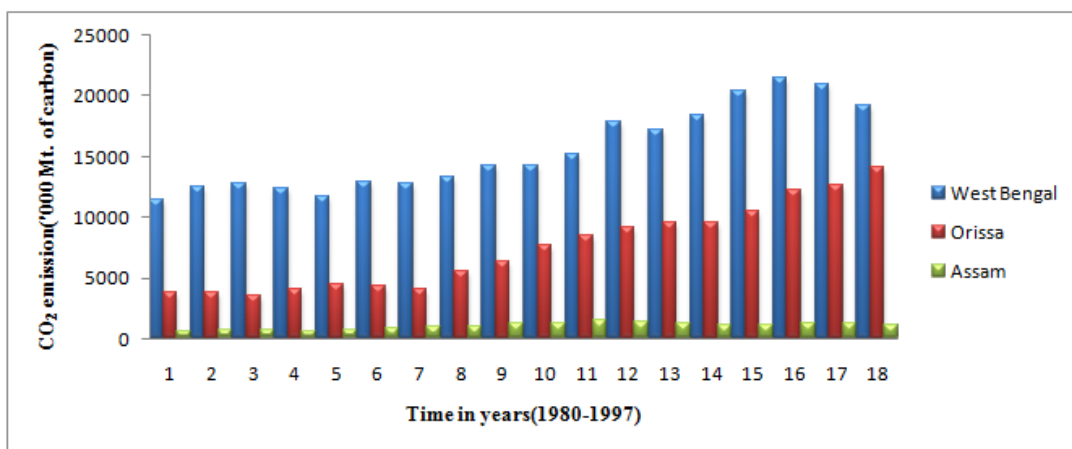
**IV. Quality Of Proposed Model And Prediction:**

For evaluating model performance, we use two statistical criteria,  $R^2$  (adjusted  $R^2$ ) and residual analysis. The coefficient of determination  $R^2$  is defined as the proportion of the total response variation that is explained by the model (Tarpey, 2000). It provides an overall measure of how well the model fits. A prediction is made for the long term evolutionary trend of CO<sub>2</sub> emission using IROC in the concerned states. The result may be helpful for better understanding about the increased or decreased emission rate of CO<sub>2</sub> into the atmosphere and the model can be readjusted to curb the emission of the said gas in near future.

**V. Result and Discussion:**

We added a comparative study diagram of CO<sub>2</sub> emission the states in the Fig. 1 so that it is easier to quantify the magnitude and characters of the emission in three different states.

The CO<sub>2</sub> emission in Orissa shows significantly increasing trend especially in the last lap of observation 1994-1997. The CO<sub>2</sub> emission in West Bengal and Assam remains steady showing good curbing of emission of CO<sub>2</sub>. The increment of CO<sub>2</sub> emission is slightly increasing and then decreasing in West Bengal and Assam, adjoining states. However, the growth in Orissa is rapid and consistent. The amount of emission in Assam in general is low compared to other states and almost steady.



**Figure 1:** Comparative bar diagram of CO<sub>2</sub> emission.

For the CO<sub>2</sub> emission in different states of India like West Bengal, Orissa, and Assam, the empirical equation of CO<sub>2</sub> utilizing the dataset of about 18 years (1987-1997) can be expressed as

$$y(\text{WB}) = 2.231086952713671 \times 10^{10} - 3.360855097517735 \times 10^7 x + 16875.309369596765x^2 - 2.824372851915935x^3 \dots\dots\dots (4a)$$

$$y(\text{Orissa}) = 1.190571282292853 \times 10^{10} - 1.790697979369143 \times 10^7 x + 8977.331426542689x^2 - 1.5001328535670677x^3 \dots\dots\dots (4b)$$

$$y(\text{Assam}) = 7.285950900904009 \times 10^8 - 1107490.508361963x + 561.0872488809084x^2 - 0.0947452586944873x^3 \dots\dots\dots (4c)$$

where x represents time in years.

The observation for the three years (1998-2000) is excluded from modelling for validation. The graphical display of the observed data and solution of the above models for CO<sub>2</sub> emission by least square method are shown in the following Fig 2 (a-c) for West Bengal, Assam and Orissa respectively which shows that CO<sub>2</sub> emissions almost possess an increasing tendency in West Bengal and Orissa and the state of Assam indicates decreasing pattern at the end of the years of study.

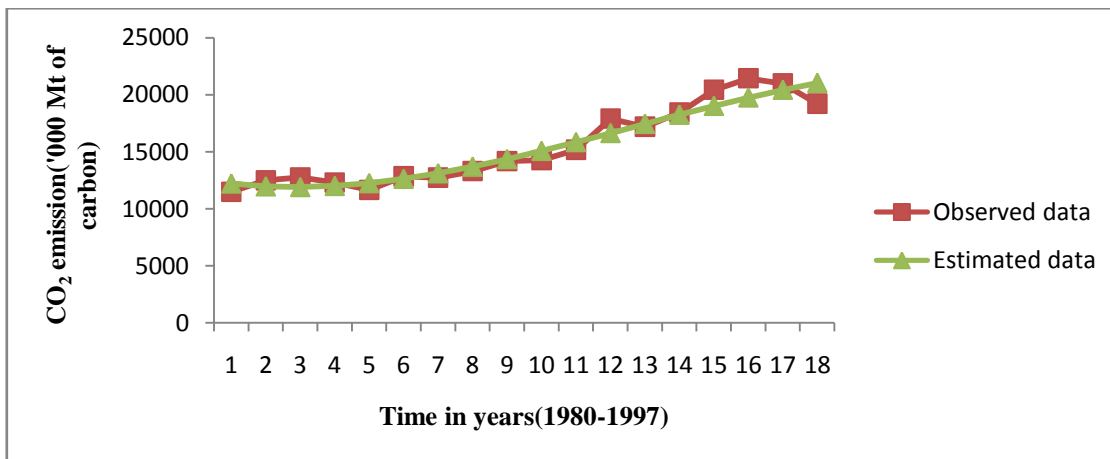


Figure 2(a): CO<sub>2</sub> emission in West Bengal.

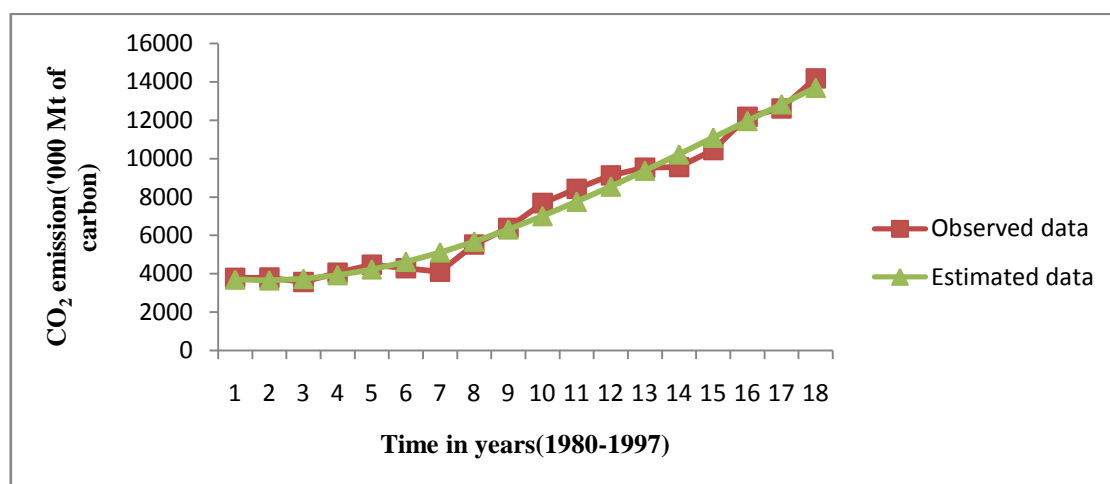


Figure 2(b): CO<sub>2</sub> emission in Orissa.

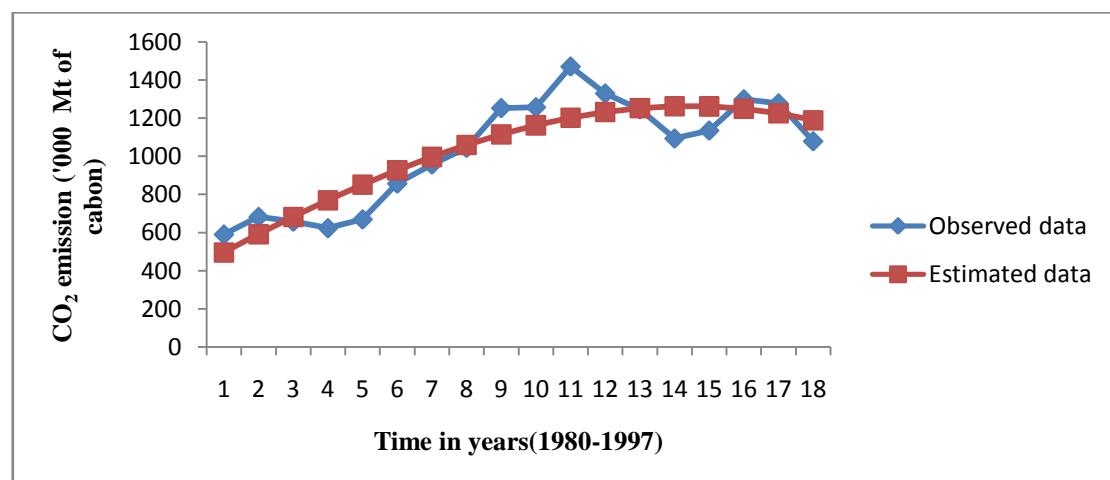


Figure 2(c): CO<sub>2</sub> emission in Assam.

From the graphs indicated in Fig 2(a-c), it is clear that CO<sub>2</sub> emission in West Bengal is highest among the states and it is followed by Orissa and Assam respectively. The goodness of the proposed model measured here by utilizing coefficient of determination  $R^2$  and  $R^2$  (adjusted). The numerical values of  $R^2$  and  $R^2$  (adjusted) for West Bengal, Orissa and Assam are shown in Table 1.

**Table 1: Statistical Evaluation Criteria for CO<sub>2</sub> emission**

States	R <sup>2</sup>	R <sup>2</sup> (adjusted)
West Bengal	0.883830	0.858936
Orissa	0.986094	0.983114
Assam	0.771123	0.722078

The numerical value of R<sup>2</sup> and R<sup>2</sup> (adjusted) reflects the fact that we have identified a good model.

**VI. Instantaneous Rate Of Change (IROC) Of CO<sub>2</sub> Emission**

IROC is a useful parameter to estimate the future emission of this green house gas. IROC of CO<sub>2</sub> emission in West Bengal, Orissa and Assam can be represented as function of time as follows

$$\frac{dy}{dx} (WB) = -3.360855097517735 \times 10^7 + 33750.61873919353x - 8.473118555747805x^2 \quad \dots\dots(5a)$$

$$\frac{dy}{dx} (Orissa) = -1.790697979369143 \times 10^7 + 17954.662853085378x - 4.500398560701203x^2 \quad \dots\dots(5b)$$

$$\frac{dy}{dx} (Assam) = -1107490.508361963 + 1122.1744977618168x - 0.2842357760834619x^2 \quad \dots\dots(5c)$$

A graphical visualization of the above equations for West Bengal, Assam and Orissa are given below

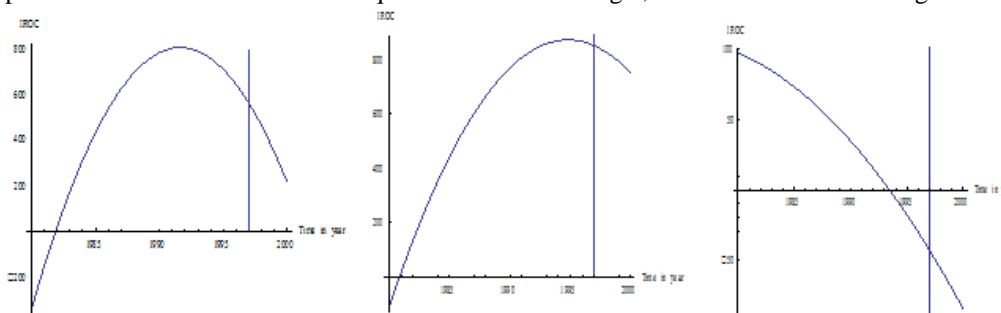


Figure 3(a): IROC of West Bengal.

Figure 3(b): IROC of Orissa.

Figure 3(c): IROC of Assam.

The graphs of IROC for West Bengal, Orissa and Assam are presented in Fig 3(a – c) to estimate the rate of change of CO<sub>2</sub> emission in the states of India for short and long term of time. In West Bengal the IROC is negative before 1982 and after that it is positive and emission grows rapidly up to 1991. For Orissa, it is negative before 1981 and after that it gradually increases and becomes maximum in 1995. After 1995 the IROC decreases gradually. In case of Assam IROC is positive before 1994 and after that it becomes negative and continues decreasing trend rapidly. Rapid increasing trend of IROC value indicating uncontrolled emission of CO<sub>2</sub> in atmosphere from different sources which may be alarming for the future. Vertical lines in the figures shows completion of 18 years of estimated IROC values for emitted CO<sub>2</sub>. We are able to predict future IROC for CO<sub>2</sub> if we notice the trend of curve after vertical line in Fig. 3(a-c) respectively from where the future emission of the gas can be done for short and long range of time. Negative value of IROC in future stands for decreasing tendency of CO<sub>2</sub> emissions year by year. In that sense, from the Fig 3(a-b) for West Bengal and Orissa have decreasing tendency in future whereas for Assam decreasing tendency persists from the beginning of the data.

**VII. Goodness Of The Model**

The observed emission and empirical emission of CO<sub>2</sub> for the year (1998-2000) of validation is presented in the following table.

**Table 2:** Observed and empirical emission of CO<sub>2</sub> in West Bengal, Orissa and Assam

State	Year	Empirical Emission('000 Mt of carbon)	Observed Emission('000 Mt of carbon)
West Bengal	1998	21554.20	17753.24
	1999	21959.50	23457.05
	2000	22239.80	23363.71
Orissa	1998	14528.00	13546.24
	1999	15339.50	15957.24
	2000	16113.20	16172.30
Assam	1998	1139.79	923.27
	1999	1076.66	1205.10
	2000	999.334	1097.00

We predict the CO<sub>2</sub> emission from the instantaneous variation of CO<sub>2</sub> emission in future from the IROC for future 4 years in West Bengal, Orissa and Assam. The empirical IROC and future emission of CO<sub>2</sub> of the said states are shown in Table 3 below.

**Table 3:** IROC and prediction of CO<sub>2</sub> emission in future

Year		2001	2003	2005	2007
IROC in future	West Bengal	61.956	-289.544	-708.825	-1195.89
	Orissa	700.238	570.372	404.503	202.630
	Assam	-99.670	-131.481	-165.566	-201.925
Emission of carbon dioxide	West Bengal	22378.40	22162.10	21175.00	19281.60
	Orissa	16839.90	18116.50	19097.40	19710.50
	Assam	907.238	676.466	379.798	12.685

From Table3, we can find out the total CO<sub>2</sub> emissions in West Bengal in 2001,2003,2005,2007 are 22378.40, 22162.10, 21175.00 and 19281.60('000 Mt of carbon) respectively. Those of for Orissa are 16839.90, 18116.50, 19097.40 and 19710.50('000 Mt of carbon) and for Assam are 907.238, 676.466, 379.798 and 12.685('000 Mt of carbon) respectively.

### VIII. Conclusion

This paper deals with the CO<sub>2</sub> gas emission in three eastern and north-eastern states in India namely West Bengal, Orissa and Assam respectively. We have developed a third degree polynomial for identifying the pattern of CO<sub>2</sub> emission in the said states using the actual data from 1980 to 1997. Moreover, we have verified some statistical tools like R<sup>2</sup> and R<sup>2</sup> (adjusted) to measure the quality of the model prediction that leads to satisfactory result. These statistical tools reflect the fact that we have identified a good model. With the value of coefficient of determination for the states of West Bengal, Orissa and Assam are 0.883830, 0.986094 and 0.771123 respectively which suggest that the model fits extremely well in the respective states. We also predicted total CO<sub>2</sub> emission in future using the model for the year 2001, 2003, 2005 and 2007 respectively. It gives a theoretical basis for the further research on the situation of CO<sub>2</sub> emission in those states of India. Lastly, our study may be inspiration for the researchers in future to identify the emission of other GHGs.

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